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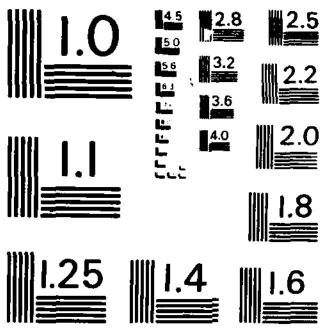


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# A CODE DIVISION MULTIPLE ACCESS COMMUNICATION SYSTEM FOR THE LOW FREQUENCY BAND

PER KRISTIAN ENGE



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In this thesis, a code division multiple access (CDMA) communication system for the low frequency (LF) channel is proposed, discussed and analyzed. This LF/CDMA scheme is similar to classical CDMA schemes in that K users share a channel by phase modulating their transmissions with signature sequences. Our LF/CDMA scheme is different in that each user's signature sequence set consists of M orthogonal sequences and thus $\log_2 M$ bits of information are transmitted by choosing among the signature sequences. Additionally, the users use r-phase modulation and our model includes an impulsive (non-Gaussian) noise source to—

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We derive a locally optimum (small signal) receiver structure for our LF/CDMA scheme. This receiver consists of a bandpass correlator followed by a sampler, a zero memory nonlinearity and M discrete time matched filter/correlators. We analyze the performance of this structure in combined multiple access, impulsive and Gaussian noise. When the noise is dominated by either its multiple access or Gaussian component, the receiver predominantly operates in the linear region of the nonlinearity and performance is similar to that of a linear receiver.

We analyze the multiuser error probability (MEP) of the linear receiver by using a Gaussian assumption to find an approximation and a characteristic function method to achieve a sharp upper bound. The approximation and the bound sometimes have substantial discrepancies and these differences are discussed.

Finally, we design two actual sequence sets for our LF/CDMA scheme. Our sequence set design strategy is to minimize the maximum magnitude of the interference that any one user's transmitter can introduce into the receiver of another user. Two sequence designs result, where one is based on additive characters and the other on multiplicative characters. These designs are analyzed and their differences are discussed.

# A CODE DIVISION MULTIPLE ACCESS COMMUNICATION SYSTEM FOR THE LOW FREQUENCY BAND

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THESIS

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1983

Urbana, Illinois

# A CODE DIVISION MULTIPLE ACCESS COMMUNICATION SYSTEM FOR THE LOW FREQUENCY BAND

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# CHAPTER 1

# INTRODUCTION

## 1.1 Overview

In this thesis, we design and analyze a code division multiple access (CDMA) communication system, which is especially suited for the low frequency (LF) band. The LF band is the portion of the frequency spectrum from 30 kHz to 300 kHz. General descriptions of propagation modes, noise models and engineering practice applicable to this band are available in Naval Shore Electronics (1972) and Watt (1967). The LF band is characterized by propagation modes, which make long range communication possible and by impulsive atmospheric noise, which is distinctly non-Gaussian in nature. Most importantly, LF communication channels have inherently narrow bandwidth and this limitation is a most important consideration when designing LF spread spectrum systems. The system, which we discuss and analyze in this thesis, is designed to make thorough use of this limited bandwidth. As such, it has many differences (as well as many similarities) with the classical spread-spectrum communication schemes, such as those described in Pursley (1977,1981); Pursley and Sarwate (1977); Pursley, Garber and Lehnert (1980); and Garber and Pursley (1981).

Our low frequency code division multiple access (LF/CDMA) communication system is similar to the classical model in the following respects. First of all, it has K users sharing a channel by phase modulating their transmissions with signature sequences. These users make no attempt at frequency separation and the scheme is asynchronous, which means that the users are not time coordinated in any way. The model also includes an additive noise source to account for atmospheric and receiver noise.

Our scheme also has the following features, which are not part of the classical models. Each user's signature sequence set consists of M orthogonal sequences and thus $\log_2 M$ bits of information are transmitted by choosing among the signature sequences. Typically, M is of the order of several hundreds. Each signature sequence is r-valued (which means r-phase modulation is used), where r may be as much as one hundred. We can consider size M sequence sets and r-phase modulation, because the long times associated with the LF band allow an increased freedom in signal set design. Furthermore, the initial carrier phase for each user is not modelled as random, because LF transmitter techniques are such that the carrier phase is fixed and stable with respect to the signal envelope. The net result is that each user produces transmitted signals with bandwidths of 5 kHz to 20 kHz and with information rates up to 1 kilobit per second.

As mentioned above, LF atmospheric noise is "impulsive", and not Gaussian. We discuss the various models, which have been used to describe the first order probability function (pdf) of impulsive noise and present an argument for using a Cauchy pdf. After discussing atmospheric noise models, we derive a locally optimum Bayes' detector for use as a receiver. This receiver contains a filter matched to the basic "chip" waveform followed by a zero memory nonlinearity (ZNL). The ZNL is in turn followed by a bank of M correlators, where each correlator corresponds to one of the possible M signature sequences. We analyze the performance of our receiver and find that the performance is a strong function of Fisher's information number, which completely depends on the statistics of the filtered noise. The analysis also shows that all LF/CDMA receivers should employ a ZNL to limit the effect of the impulsive noise. Additionally, if the multiple access or Gaussian noise is strong, then the performance of this nonlinear receiver closely resembles the performance of a linear receiver operating in the combined multiple access and Gaussian noise (without impulsive noise.)

Consequently, we consider a linear receiver (without impulsive noise), which we analyze using a characteristic function method (Geraniotis and Pursley (1982)). The characteristic function approach yields a bound for the receiver's probability of error. An error probability approximation can be obtained by assuming the multiple-access interference is approximately Gaussian and using the well known results for coherent M-ary communication in Gaussian noise. The results from the characteristic function analysis can differ quite significantly from those based on the Gaussian assumption, and we provide a brief discussion of these discrepancies. For most of our linear analysis, we consider the user codes to be random cosets of an orthogonal code.

Following the system analysis portion of the thesis, we design two specific sequence sets, which are well suited for our LF/CDMA communication scheme. Both designs have upper bounds for the maximum value of the magnitude of the multiple access interference. These interference bounds are determined through the use of a partial sum theorem. Even though our main interest in this thesis is multiple access interference, skywave (specular multipath) and signal acquisition performance could be major issues in the design of any LF communication systems. Consequently, both of our sequence sets are also designed to have good skywave and acquisition performance.

One of our sequence set designs is based on additive characters and the other is based on multiplicative characters. The main differences between the two designs is the number of required phases and number of accommodated users. The additive character design accommodates N users, but requires N phases, where N is the sequence length. The multiplicative character design uses many fewer than N phases, but consequently accommodates many fewer than N users.

In the remainder of this introductory chapter, we will describe spread spectrum multiple access research, the thesis outline, and the contributions of the thesis.

## 1.2 Spread Spectrum Multiple Access Communication

Spread spectrum systems are those where the transmitted signal occupies much greater bandwidth than the data signal itself. Bandspreading is commonly achieved either by frequency hopping (FH), where the center frequency of the transmitted signal is varied, or direct sequence (DS) which varies the phase. Spread spectrum systems use greater bandwidth, because the transmitted signal has been carefully designed to achieve specialized system goals, some of which are:

- high resolution ranging
- multipath rejection
- signal hiding
- multiple access capability.

The last of these goals is of greatest interest to us and we now consider a direct sequence spread spectrum multiple access system.

Some spread spectrum systems use phase or direct sequence modulation to achieve multiple access communication. Such systems are called code division multiple access (CDMA) systems and one such scheme has been modelled and analyzed by Pursley (1977). In Pursley's model, each transmitter has a single binary signature sequence and phase modulates its carrier with the sequence or its negative depending on the data. Each receiver contains a filter matched to the carrier modulated by the desired sequence and uses the output of the filter as its decision statistic. The receiver decides the value of the data bit based on the sign of the decision statistic. However, the receiver's decision is not perfectly reliable, because of noise in the channel (which is assumed to be an independent additive white Gaussian process) and because of signals from each of the competing users, who share the channel. Each of these users contributes an interference term to the decision statistic and the sum of these terms is known as multiple access interference. Each of the interference terms is a function of the following variables:

- the time of arrival delay of the competing users' signal with respect to the desired signal
- the phase offset of the competing users' transmitters relative to the desired signals' transmitter
- the competing users' data.

In general, these variables may be characterized in different ways to fit the system under consideration and the issues at hand. In Pursley's 1977 paper, two different analyses based on different characterizations of the underlying variables are presented and are quickly reviewed below.

Worst case analysis considers the delay, phase offset, and data variables to be deterministic and finds the value of these variables for which the multiple access interference takes its maximum magnitude. This approach results in an upper bound on the receiver's probability of error and provides a mini-max design criterion for the selection of signature sequences. This mini-max sequence design criterion is further investigated in Pursley and Sarwate (1977), Sarwate (1979), and Sarwate and Pursley (1980). The deterministic analysis also shows that the worst case performance is independent of the shape of the chip waveform (the chip waveform is the basic signal, which the transmitter phase modulates and transmits). Worst case analyses guarantee a performance minimum under the worst of conditions. On the other hand, the worst of conditions generally occurs so infrequently that no estimate of the average performance of the system is obtained.

For this reason, average system performance is estimated by considering each interference term to be a function of independent random variables. This approach implies that the multiple access interference is also a random variable and the mean and variance of the multiple access interference may be calculated. The mean is zero and the variance may be used to calculate the receiver's signal to noise ratio. The signal to noise ratio may, in turn, be used to calculate the receiver's probability of error if the multiple access interference is assumed to have a Gaussian distribution. This Gaussian assumption gives reasonably accurate estimates of the actual probability of error (Pursley et al. (1982), Geraniotis and Pursley (1982)). In Pursley's model, the average performance analysis yields signal to noise ratios (and consequently probability of error estimates) as a function of the mean square correlation parameters of the signal sets. These parameters are explored as alternate sequence design criteria in Pursley and Sarwate (1977) and Sarwate and Pursley (1980). Additionally, the average analysis shows that the signal to noise ratio is a function of the mean square correlation parameters of the chip waveforms. These parameters have been used as chip waveform design guides by Lehnert (1981).

Random sequence analysis is also explored as a CDMA system design tool by Pursley (1977) and Roefs and Pursley (1976). Here, the K signature sequences are considered to be mutually independent random sequences and each sequence is considered to be a sequence of independent identically distributed binary random variables. The expected value of the mean square correlation parameters for the random sequences may then be readily calculated. In this way, random sequence analysis provides a straight forward estimate of system performance (signal to noise ratio and probability of error) without requiring any actual sequence design.

Recently, the average probability of error in CDMA systems has been calculated via a characteristic function method (Geraniotis and Pursley (1982) and Geraniotis (1982)). This approach eliminates the uncertainty due to the Gaussian approximation and has shown that the latter does sometimes lead to appreciable errors. In Geraniotis and Pursley (1982), the characteristic function method is applied to binary and quaternary CDMA systems, operating in additive white Gaussian noise. Results are given for various signature sequences (m-sequences and Gold sequences) and a collection of chip waveforms. In Geraniotis (1982), the method is used on direct sequence and frequency hopped SSMA communication systems, which suffer fading as well as additive noise. In this latter work, specific signature sequences are not considered, because random sequence analysis is used.

The role of all these CDMA design tools in this thesis will be discussed shortly.

## 1.3 Thesis Outline

In Chapter 2 of the thesis, we describe atmospheric noise and consider appropriate nonlinear receivers. Specifically in Section 2.2, we introduce our LF/CDMA model and in Section 2.3, we describe LF atmospheric noise and our choice for modelling its first order probability distribution function. In Section 2.4, we derive a locally optimum Bayes' detector (LOBD) of M signals and in Section 2.5 we approximate the performance of this structure in combined atmospheric, Gaussian, and multiple access noise.

In Chapter 3, we consider linear receivers and calculate average multiple access performance. In Section 3.2, we begin by considering single user systems and in Section 3.3, we present the random coding ideas which we will use in our multiuser analysis. Section 3.4 is devoted to multiuser analysis and both the Gaussian approximation and the characteristic function method are used. This section compares the results of these two approaches and also considers some alternate signalling schemes.

In Chapter 4, we design specific sequence sets for our LF/CDMA scheme by using a minimax approach. In Section 4.2, we begin by developing a minimax sequence design criterion by performing a worst case multiple access interference analysis.

The next section develops this criterion through application of a partial sum theorem. In Sections 4.4 and 4.5, we find additional sequence design criteria for multipath and signal acquisition respectively. In Section 4.6, we describe additive characters in general and use them to design sequences for our LF/CDMA scheme. In Section 4.7, we describe multiplicative characters and use them to design LF/CDMA sequences.

In Chapter 5, we summarize the main conclusions of the thesis.

## 1.4 Contributions of the Thesis

The first contribution of the thesis is the low frequency CDMA signalling scheme itself. Specifically, each user has M orthogonal signature sequences and consequently $\log_2 M$ bits of information are communicated by choosing among the sequences. Additionally, the sequences are r-valued, which means that each user is employing r-phase modulation.

The second major contribution of the thesis is the consideration of impulsive atmospheric noise. This noise causes us to derive a locally optimum Bayes' detector of M signals, which includes a zero memory nonlinearity. We analyze the performance of this nonlinear receiver. We establish the similarity between the performance of the nonlinear receiver operating in combined Cauchy, Gaussian, and multiple access noise and the performance of a linear receiver operating in combined Gaussian and multiple access noise.

Another contribution of the thesis is to extend the use of the characteristic function method to the analysis of our LF/CDMA signalling scheme. We add to the information regarding the shortcomings of the Gaussian approximation for CDMA performance analysis.

Another contribution of the thesis is the application of a partial sum theorem in the pursuit of a minimax sequence design criterion. Finally, we use the minimax criterion to synthesize 2 sequence designs for our LF/CDMA scheme. These designs provide K sets of M orthogonal sequences. Both designs have "good" multiple access, skywave, and signal acquisition properties.

# CHAPTER 2

# SYSTEM ANALYSIS PART I:

# RECEIVERS WITH NONLINEARITIES

## 2.1 Introduction

The goal of Chapters 2 and 3 is to analyze our low frequency CDMA communication system and present equations which will describe the basic system tradeoffs. To this end, we begin this chapter by describing our LF/CDMA communication system design model. Next, we describe LF atmospheric noise and describe a noise model, which is appropriate for the LF channel. We then derive a locally optimum (small signal) Bayes detector for M signals and approximate the performance of this detector. The most interesting feature of the locally optimum Bayes detector (LOBD) is a zero memory nonlinearity (ZNL) and the performance analysis shows that the performance of the overall detector is a strong function of Fisher's information number. Consequently, we conclude Chapter 2 by presenting and discussing curves, which describe Fisher's information number. In Chapter 3, we continue our system analysis by considering a linear analysis.

## 2.2 Low Frequency CDMA Communication System Design Model

In this section, our CDMA design model is presented and discussed in detail. As shown in Figure 2.1, every user has a data alphabet of size M, and corresponding to each element of this data alphabet is a length N signature or bandspreading sequence. The sequence elements are $r^{th}$ order roots of unity;

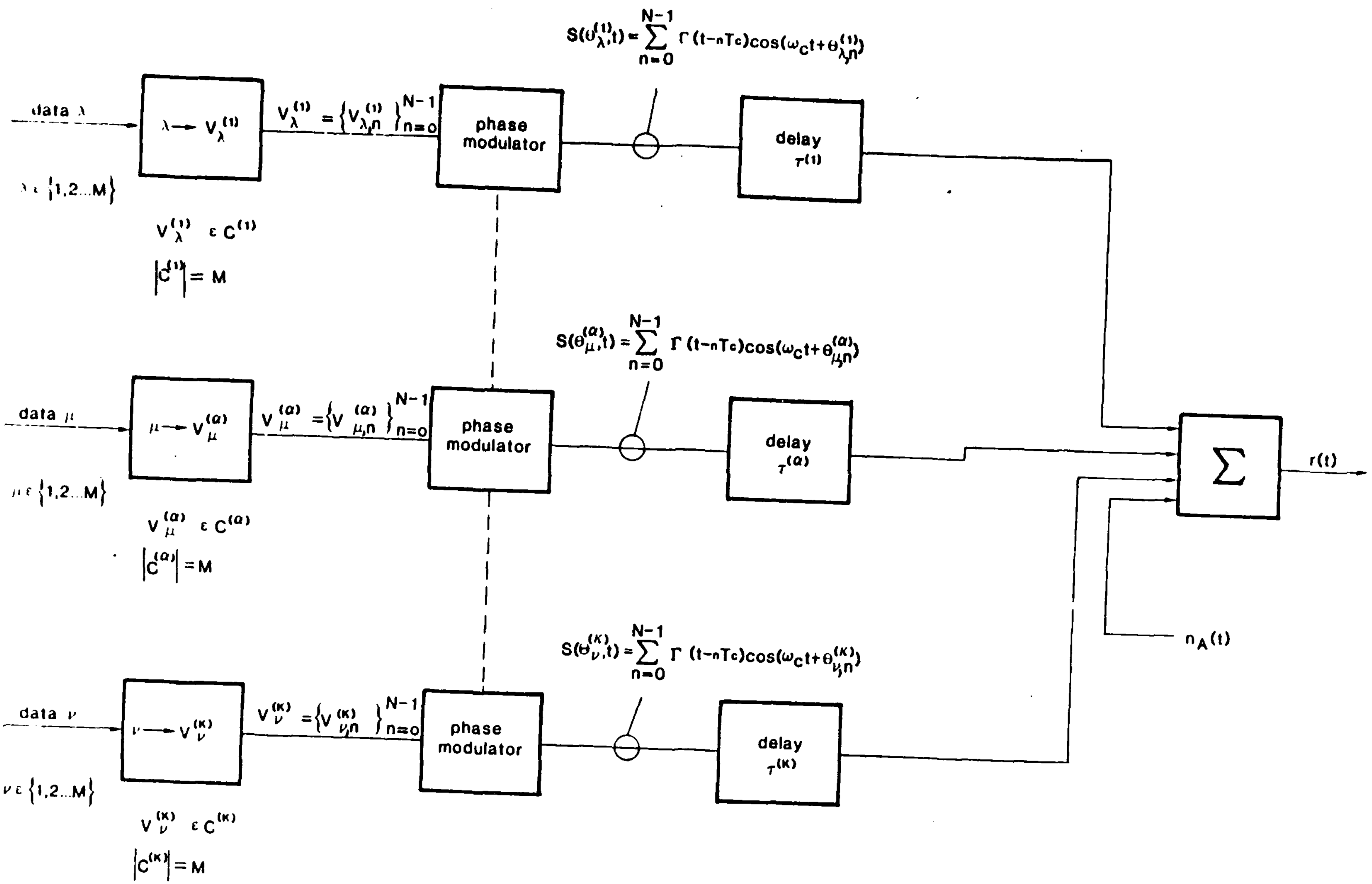


Figure 2.1 LF/CDMA Communication System

hence, r phases are used to modulate the carrier. The sequence set or code for the $\alpha^{th}$ user is denoted by $C^{(\alpha)}$, where

$$C^{(\alpha)}=\{v_1^{(\alpha)} \ \ldots \ v_M^{(\alpha)}\} \tag{2.1}$$

is a set of vectors.

$$v^{(\alpha)}=[v_{\lambda,0}^{(\alpha)} \ \ldots \ v_{\lambda,N-1}^{(\alpha)}] \qquad \lambda=1,2,\ldots M \tag{2.2}$$

The sequence corresponding to the current data is used to phase modulate the transmitted carrier. Therefore, the signal transmitted by the $\alpha^{th}$ transmitter during the interval $t\varepsilon[\emptyset,NT_c)$ is

$$s(v_\lambda^{(\alpha)},t)=\mathrm{Re}\{\sum_{n=0}^{N-1} v_{\lambda,n}^{(\alpha)}\ \Gamma(t-nT_c)\ \exp j\omega_c t\} \tag{2.3}$$

$$=\sum_{n=0}^{N-1}\Gamma(t-nT_c)\ \cos(\omega_c t+\theta_{\lambda,n}^{(\alpha)})$$

where

$$v_{\lambda,n}^{(\alpha)}=\exp j\theta_{\lambda,n}^{(\alpha)} \qquad \lambda\varepsilon\{1,2\ldots M\} \tag{2.4}$$

$$n\varepsilon\{0,1\ldots N-1\}$$

$\Gamma(t)$ is the chip waveform common to all users. It is of duration $T_c$ and is the envelope of the modulated carrier. In LF applications, $\Gamma(t)$ need not be a constant envelope signal. The energy of the chip waveform is

$$\varepsilon_\Gamma=\int_0^{T_c} \Gamma^2(t)dt \tag{2.5}$$

and, consequently, the energy of the signal is

$$\varepsilon_s=\int_0^{NT_c} |s^{(\alpha)}(t)|^2 dt=N\varepsilon_{\Gamma}/2 \tag{2.6}$$

The carrier frequency shared by all users is $\omega_c$ radians per second. Note that the phase of the carrier during the $n^{th}$ chip interval $[nT_c,(n+1)T_c)$ is controlled by the $n^{th}$ sequence element only. With LF transmission, no additional phase offset needs to be modelled. In other words, two signals will be in phase if their relative delay is zero and they are identically modulated. The bandwidth expansion factor of a user's sequence set is the ratio of the transmitted bandwidth to the information rate (BW/R). For a pulse of duration $T_c$, the first null to first null transmitted bandwidth (BW) is approximately $2/T_c$. Therefore, the bandwidth expansion factor is given by

$$BW/R=2N/\log_2 M \tag{2.7}$$

Additionally, the transmitted energy per information bit is

$$\varepsilon_b=N\varepsilon_{\Gamma}/2\log_2 M \tag{2.8}$$

In the channel, each signal is delayed by an amount $\tau^{(\alpha)}$ and is corrupted by atmospheric noise ($n_A(t)$). We denote the vector of signal advances by

$$\tau=[\tau^{(1)}\ldots\tau^{(K)}] \tag{2.9}$$

In this thesis, we do not include individual attenuation terms for each signal, but these could easily be introduced. Without loss of generality, we study a receiver for $C^{(1)}$ and we set $\tau^{(1)}$ equal to zero, since we assume the receiver is synchronized. The received signal is denoted by

$$r(t)=s(v_{\lambda}^{(1)},t)+n(t) \qquad t\varepsilon[0,NT_C) \tag{2.10}$$

where the data word sent is $\lambda$ . If a single user is signalling,

$$n(t)=n_A(t) \tag{2.11}$$

where $n_A(t)$ is the LF noise process observed at the input of the receiver. If more than one user is transmitting,

$$n(t)=n_A(t)+\sum_{\beta=2}^{K} s(b^{(\beta)},t+\tau^{(\beta)}) \qquad t\varepsilon[0,NT_C) \tag{2.12}$$

In this equation, the interfering signals are shifted in time by $\tau^{(\beta)}$ relative to the reception of the desired signal. Consequently, two codewords from $C^{(\beta)}$ contribute to the modulation of the signal from the $\beta^{th}$ transmitter, during the interval $[0,NT_C)$. This is denoted through the vector $b^{(\beta)}$ (a 2N-tuple), which is a concatenation of the first and second sequences received from the $\beta^{th}$ user during the interval $[0,NT_C)$. We may write

$$b^{(\beta)}=[u^{(\beta)}|v^{(\beta)}] \qquad u^{(\beta)},v^{(\beta)}\varepsilon C^{(\beta)} \tag{2.13}$$

This notation allows us to write

$$s(b^{(\beta)},t+\tau^{(\beta)})=\sum_{n=0}^{N-1} \mathrm{Re}\{b_{\ell+n}^{(\beta)}\ \Gamma(t+\tau^{(\beta)}-nT_c)\exp j\omega_c(t+\tau^{(\beta)})\} \tag{2.14}$$

where

$$\ell T_c \leq \tau^{(\beta)} < (\ell+1)T_c \tag{2.15}$$

Finally, we denote

$$b_n^{(\beta)}=\exp j\theta_{b,n}^{(\beta)} \tag{2.16}$$

In the next section, we take a more detailed look at the atmospheric noise term $n_A(t)$.

## 2.3 Atmospheric Noise

LF atmospheric noise is predominantly generated by lightning strokes. This noise has much greater power than galactic or receiver noise sources, hence, a LF noise process appears as a low power Gaussian process punctuated by high amplitude transients. Consequently, the atmospheric noise process may be expressed as

$$n_A(t)=n_G(t)+n_C(t) \tag{2.17}$$

where $n_G(t)$ is a Gaussian process, and $n_C(t)$ is an impulsive noise process. The Gaussian component is due to the combined contribution of noise sources which are either weak or distant plus thermal noise in the receiver. In our receiver, the noise and signals pass through a matched filter and are sampled. The portion of this sample due to noise is denoted

$$\eta_A = \eta_C + \eta_G \tag{2.18}$$

where $\eta_C$ is the impulsive component and where $\eta_G$ has a Gaussian distribution. The "impulsiveness" of $\eta_C$ is reflected in its voltage deviation

$$v_d = E\{(\text{envelope}(n_A(t))^2)\}^{\frac{1}{2}} / E\{\text{envelope}(n_A(t))\} \tag{2.19}$$

The voltage deviation of band pass atmospheric noise is plotted in Figure 2.2 as a function of center frequency and bandwidth. For wide bandwidths, the deviation is large, because a small number of atmospheric noise events will dominate the output of the filter. On the other hand, for narrow bandwidths, the deviation decreases and approaches the value for a Gaussian process. This is because the filter's time constant is large compared to the interarrival time of the impulsive events and the contribution of any one impulse cannot dominate. In Figure 2.2, the deviation curves for two sets of bandwidth are shown. The upper curves correspond to bandwidths that would be typical of our receiver's front end and the lower curves correspond to the post processing bandwidth when sequence lengths near 1000 are used. A more thorough investigation of the noise statistics is required to be able to predict signal detector performance.

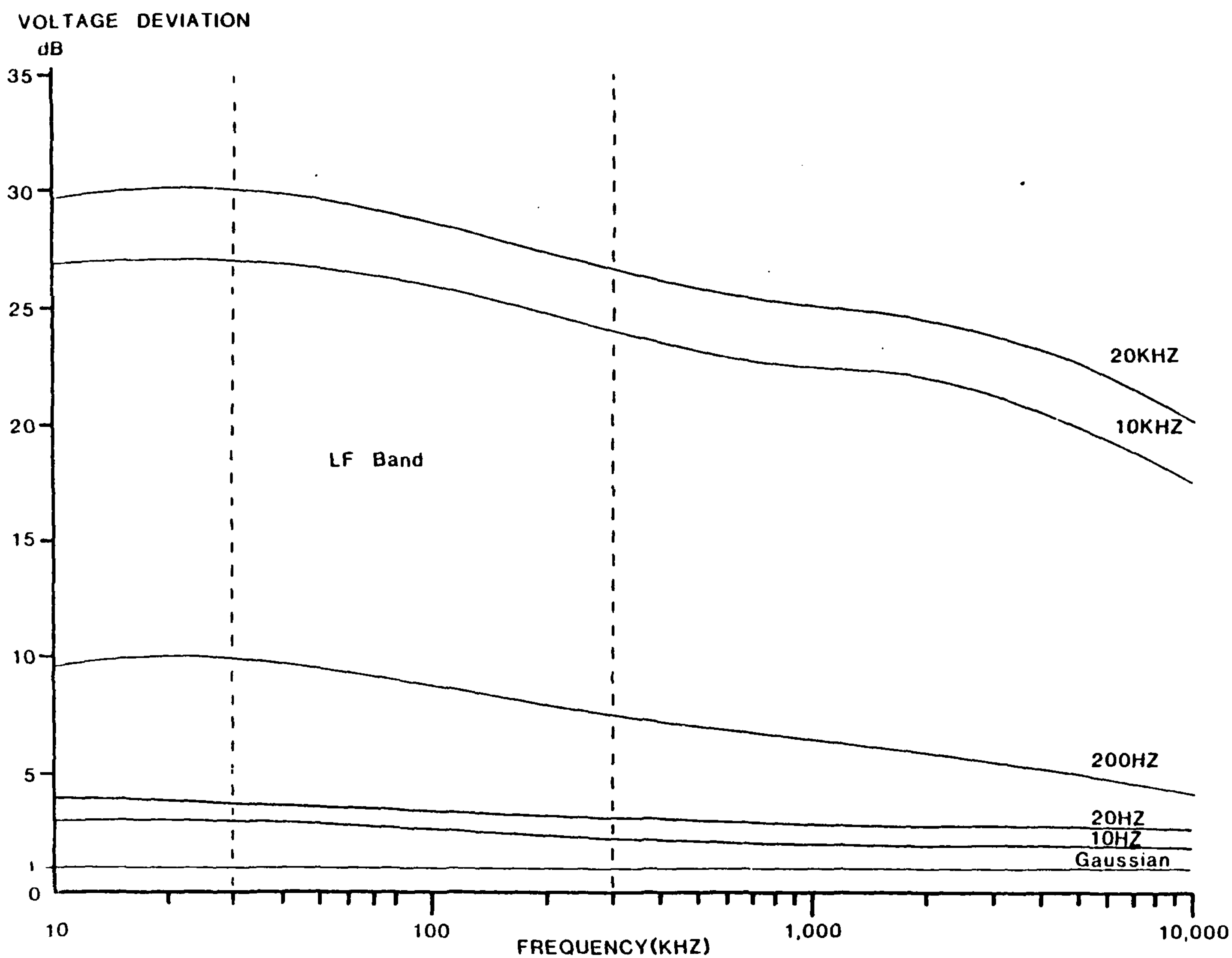


Figure 2.2 Voltage Deviation versus Frequency

Atmospheric noise has been characterized through its first order probability density function (pdf) in a number of ways in the recent literature. The noise pdf has been treated as a generalized Cauchy distribution by Hall (1966) and Feldman (1972). Also, a log normal pdf has been used by Omura and Shaft (1971). However, we will review the results of Middleton (1974, 1976, 1977, 1979a, 1979b) which are attractive, because the mathematical <u>form</u> of his results does not change with changing physical conditions. However, the <u>parameters</u> of the model are explicit functions of the underlying physical mechanisms (physical source distributions, noise source waveforms, propagation types, etc.). This means that the important parameters of the model may be readily measured. It also means that the connection between analytic simplifications and the real world may be established. Middleton's noise model is limited to the description of noise after it has been processed by a narrowband filter. (In this case, this means that the receiver front end bandwidth must be less than one fifth of the center frequency.) The other model limitation is that the noise must only be statistically related to the receiver. (Deterministic or completely known interference must be treated in another fashion.)

Three classes of noise are defined by Middleton. Class A noise is where the unfiltered noise has a power spectral density (psd) roughly equal to or less than the receiver front end bandwidth. Class B noise is where the unfiltered noise process has a psd substantially wider than the receiver front end, and class C noise is a sum of a class A and a class B noise process. The atmospheric noise, which interests us, is clearly a class B type noise process. For all three noise types, Middleton has developed a characteristic function, envelope exceedance probability, envelope pdf and envelope moment expression. All three of these expressions come in pairs for class B noise, because one expression is required to describe the small envelope noise behavior and another is required for the large envelope behavior. For Middleton's full class B model, six parameters are involved; however Spaulding (1982) has shown that for signal detection work in atmospheric noise, a simplified version of the model is quite satisfactory. In other words, we will only consider the small envelope forms of the class B model. The small envelope pdf for class B noise is:

$$p_{\eta_c}(z) = \frac{\exp(-z^2/k_1)}{\pi\sqrt{k_1}} \cdot \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} A_s^m \, \Gamma_1((ms+1)/2) \; {}_1F_1(-ms/2, \tfrac{1}{2}, z^2/k_1) \tag{2.20}$$

where $\Gamma_1$ is the Gamma function and ${}_1F_1$ is a confluent hypergeometric function described by Middleton (1976). The pdf has three parameters: s, $A_s$ and $k_1$, where s and $A_s$ are intimately linked to the physics of the noise process, and $k_1$ is a normalization parameter such that the variance of the process envelope is unity. The variable s is called the spatial density propagation parameter and describes the spatial distribution of the noise sources as well as the noise source to receiver propagation law. This parameter is limited to the values $0<s<2$ in Middleton's analysis, where he states that these values cover most of the practical cases. $A_s$ is the effective impulsive index of the noise and is a function of the impulsive index of the noise ($A_B$) as well as s. The impulsive index ($A_B$) is equal to the mean number of interference events per second times the mean duration of an event, where the duration is measured after the front end filter. As such, $A_B$ is a measure of the temporal overlap or density of the noise. As the mean number of interfering events per second increases, $A_B$ and $A_s$ increase and the noise pdf resembles a Gaussian pdf as might be expected, because of the central limit theorem.

For s = 1, Middleton's class B pdf may be approximately simplified (Middleton (1976)) to a Cauchy pdf

$$p_{\eta_c}(z)=\frac{2A_1\sqrt{k_1}}{\pi(4z^2+k_1A_1^2)} \tag{2.21}$$

with effective impulsive index $A_1$, and normalization $k_1$. A revealing expression for $A_1$ is

$$A_1=4\sqrt{A_B}/v_d \tag{2.22}$$

where $v_d$ is the voltage deviation given by Equation (2.19) and $A_B$ is the impulsive index described above.

In conclusion, the pdf of Equation (2.21) is not general enough to describe all varieties of atmospheric noise, but it is a conservative choice for the results we pursue here and consequently we will use it in the remainder of this chapter. We now turn our attention to the design and analysis of locally optimum signal detectors.

## 2.4 A Locally Optimum Bayes' Detector of M Signals

As mentioned in Section 2.3, low frequency atmospheric noise has a distinctly non Gaussian nature. Specifically, samples of atmospheric noise after front end filtering may be denoted

$$\eta_A=\eta_C+\eta_G \tag{2.23}$$

where $\eta_C$ is the predominant component and has a Cauchy first order pdf. The performance of a signal detector can be a strong function of the "match" between the assumed noise model and the actual noise, consequently, we wish to investigate which signal detection scheme is optimum for our noise model. Our optimized detector should give better performance than the optimum Gaussian noise detector operating in Gaussian noise, because Gaussian noise impairs detector performance more than any other noise type (for a given noise variance). The analysis of signal detection in non-Gaussian noise is a widely studied subject and some well known results are published in Miller and Thomas (1972), Spaulding and Middleton (1977), Spaulding (1982), Omura and Shaft (1971), and Lu and Eisenstein (1981).

In Spaulding and Middleton (1977), the optimum coherent detection of binary signals in class A impulsive noise is discussed in three main parts. First, the optimum Bayes' decision strategy is derived for class A noise and an average probability of error bound derived. Second, the performance of sub-optimum matched filter detectors in class A noise is calculated. In the third section, the optimum strategy derived in the first part is simplified through the use of a small signal assumption. This simplification is important, because the rather complex globally optimum test is reduced to a matched filter preceded by a zero memory nonlinearity as derived earlier by Miller and Thomas (1972). This theoretically derived ZNL/matched filter structure is interesting, because it resembles earlier intuitively derived receiver designs. The intuitive receiver designs for impulsive noise were based on the observation that limiting the amplitude of the received signal will improve signal detectability, because the majority of the noise power is contained in high amplitude, short duration transients.

The small signal assumption can be used to derive locally optimum detectors for signals imbedded in any variety of noise process and it can be used to approximately analyze the performance of these detectors. Indeed, this assumption has been used to derive and analyze detectors for signals corrupted by class B noise processes (Spaulding, 1982). In this section, we will use it for our model where the signal is degraded by additive impulsive noise, additive Gaussian noise and multiple access interference. As such, the small signal assumption is well suited for use with our model where the desired signal is generally small compared to the sum of the degrading signals.

The locally optimum (small signal) Bayes detector for our model is now derived and its performance approximated. A sampled data receiver is assumed where the sampler is preceded by a correlation receiver which also serves to reduce out of band interference and reduce dynamic range problems in analog to digital conversion. The local signal in the receiver is the carrier (with zero phase shift) modulated by the chip waveform (i.e., $\Gamma(t)\cos\omega_c t$). This situation is shown in Figure 2.3, where the received signal is

$$r(t)=s^{(1)}(t)+n(t) \tag{2.24}$$

The sampled output of the receiver at time $t = nT_c$ is

$$R_n=\int_{(n-1)T_c}^{nT_c} n(t)\Gamma(t)\cos\omega_c t\ dt+\int_{(n-1)T_c}^{nT_c} s^{(1)}(t)\Gamma(t)\cos\omega_c t\ dt \tag{2.25}$$

$$=\eta_n+(\mathcal{E}_\Gamma/2)\cos\theta_{\lambda,n}^{(1)}$$

where $\eta_n$ is the result of the atmospheric noise plus multiple access interference. The set $\{(R_n)\}$ are the observations from which our detector must make an optimal decision.

The Bayes decision criterion leads to a likelihood ratio test (LRT), and when the signals are equally likely and symmetric cost assignments are made, the LRT is

$$\text{choose } \lambda \leftrightarrow \quad p_{R|\lambda}(x|\lambda) \geq p_{R|\mu}(x|\mu) \tag{2.26}$$

We denote

$$R=(R_0 \;\dots\; R_{N-1}) \tag{2.27}$$

$$\eta=(\eta_0 \;\dots\; \eta_{N-1}) \tag{2.28}$$

$$S^{(1)}=(\tfrac{1}{2}\varepsilon_\Gamma)(\cos\theta^{(1)}_{\lambda,0} \;\dots\; \cos\theta^{(1)}_{\lambda,N-1}) \tag{2.29}$$

as the observation, noise and signal vectors, respectively. We now observe that

$$p_{R|\lambda}(x|\lambda)=p_\eta(x-S^{(1)}_\lambda) \tag{2.30}$$

which we approximate near $.5\varepsilon_\Gamma=0$, for threshold signals as follows

$$p_\eta(x-S^{(1)}_\lambda) \approx p_\eta(x) \tag{2.31}$$

$$-\sum_{n=0}^{N-1} \frac{\partial p_\eta(x)}{\partial x} \; (\tfrac{1}{2}\varepsilon_\Gamma)\cos\theta^{(1)}_{\lambda,n}$$

Assuming independent noise samples, along with manipulating Equations (2.26) and (2.31) yields an equivalent LRT

$$\text{choose } \lambda \leftrightarrow \; \sum_{n=0}^{N-1} \frac{d}{dR_n} \; \log p_{\eta_n}(R_n) \; \cos\theta^{(1)}_{\lambda,n} \leq \tag{2.32}$$

$$\sum_{n=0}^{N-1} \frac{d}{dR_n} \; \log p_{\eta_n}(R_n) \; \cos\theta^{(1)}_{\mu,n} \qquad \forall \mu$$

A receiver with this LOBD is shown in Figure 2.3, where a zero

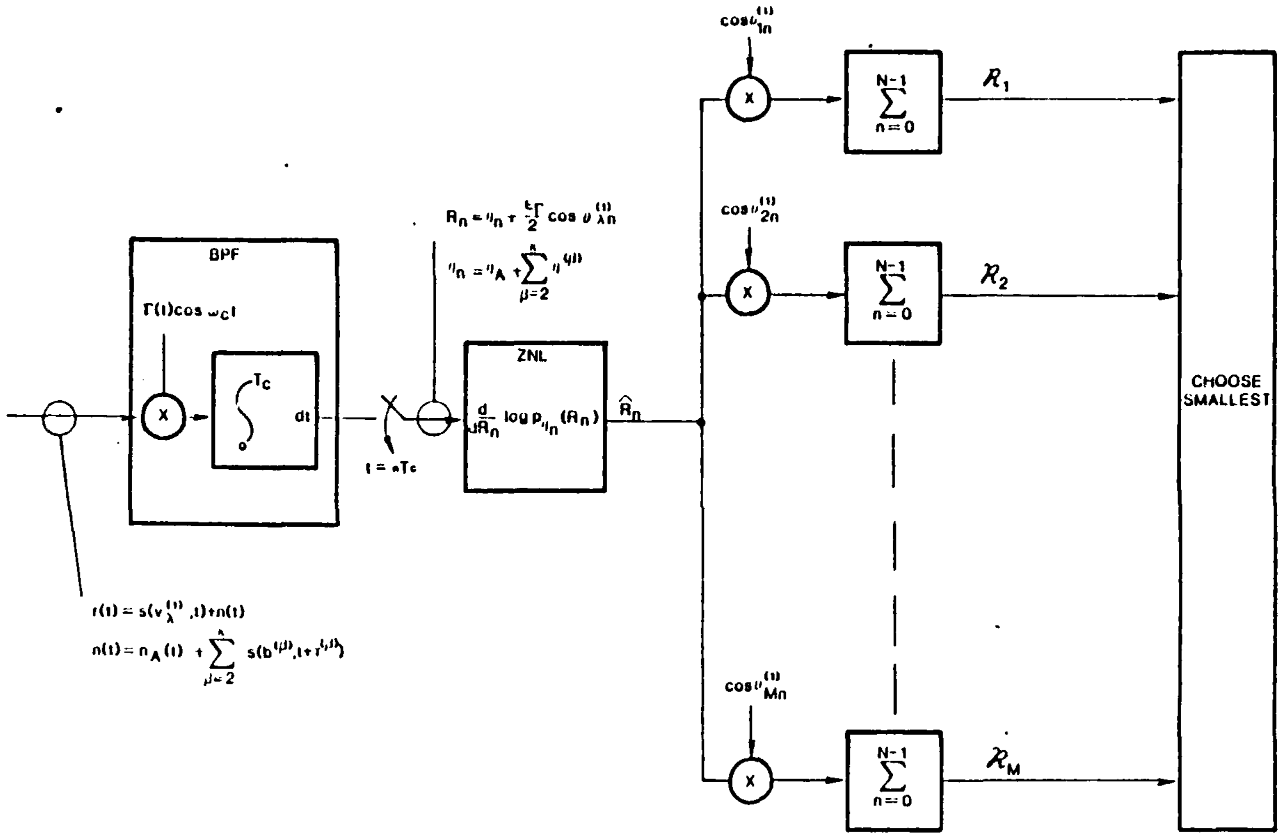


Figure 2.3 A Locally Optimum Detector of M Signals

memory nonlinearity maps the samples $R_n$ into $\hat{R}_n$. The equation for the zero memory nonlinearity (ZNL) is

$$\hat{R}_n = \frac{d}{dR_n} \log p_{\eta_n}(R_n) \tag{2.33}$$

The $\hat{R}_n$ are then processed by M discrete time correlators each matched to one of the M signals. The receiver decides which data word was sent by choosing the data word that corresponds to the correlator with the smallest output.

We now approximately analyze the performance of the LOBD, by defining the signal to noise ratio as follows.

$$\mathrm{SNR}(\mu,\lambda) = \frac{E\{\mathcal{R}_\mu|\lambda\}-E\{\mathcal{R}_\lambda|\lambda\}}{(\mathrm{Var}(\mathcal{R}_\mu|\lambda)+\mathrm{Var}(\mathcal{R}_\lambda|\lambda))^{1/2}} \tag{2.34}$$

If the $\eta_n$ are Gaussian (or the $\mathcal{R}_\mu$ and $\mathcal{R}_\lambda$ are uncorrelated and Gaussian), then this definition is especially appealing. For the Gaussian case, the probability of codeword error can be union bounded as follows

$$\Pr(\varepsilon) \leq (1/M) \sum_{\lambda=1}^{M} \sum_{\mu\neq\lambda} Q(\mathrm{SNR}) \tag{2.35}$$

where

$$Q(z)=\frac{1}{\sqrt{2\pi}} \int_z^\infty \exp(-y^2/2)\, dy \tag{2.36}$$

The moments of $\mathcal{R}_\lambda$ can be found as functions of the moments of $\hat{R}_n \cos\theta^{(1)}_{\lambda,n}$

$$E\{\hat{R}_n \cos\theta^{(1)}_{\lambda,n} | \lambda \text{ sent}\} = \tag{2.37}$$

$$E_c^{(1)}\{\int_{-\infty}^{\infty} \hat{R}_n \cos\theta^{(1)}_{\lambda,n}\, p_{\eta_n}(R_n - S_{\lambda,n})\, dR_n\}$$

In Equation (2.37), $E_c^{(1)}$ denotes expectation with respect to the phase of the $n^{th}$ chip ($\theta^{(1)}_{\lambda,n}$). In what follows, this phase is approximated as a uniform random variable on $[0,2\pi)$, because the number of phases (r) in our LF/CDMA system is typically quite large. This approximation is further discussed in Section 3.3. Employing the small signal approximation for $p_{\eta_n}(R_n)$ yields

$$E\{\hat{R}_n \cos\theta^{(1)}_{\lambda,n} | \lambda \text{ sent}\} = -E_c^{(1)}\{(\cos\theta^{(1)}_{\lambda,n})^2\, L\, \tfrac{1}{2}\varepsilon_\Gamma\} \tag{2.38}$$

$$= -\tfrac{1}{4} L \varepsilon_\Gamma$$

where

$$L = \int_{-\infty}^{\infty} \frac{\left(\frac{d}{dx}\, p_{\eta_n}(x)\right)^2}{p_{\eta_n}(x)}\, dx \tag{2.39}$$

The quantity L is known as Fisher's information number. Additionally

$$E\{\hat{R}_n \cos\theta^{(1)}_{\lambda,n} | \lambda \text{ not sent}\} = 0 \tag{2.40}$$

To find the variance, the following moments are considered

$$E\{(\hat{R}_n \cos\theta_{\lambda,n}^{(1)})^2 | \lambda \text{ sent}\} = \tag{2.41}$$

$$E_c^{(1)} \int_{-\infty}^{\infty} (\hat{R}_n)^2 (\cos\theta_{\lambda,n}^{(1)})^2 \, p_{\eta_n}(R_n - S_{\lambda,n}) \, dR_n = E_c^{(1)} (\cos\theta_{\lambda,n}^{(1)})^2 L = \tfrac{1}{2}L$$

$$E\{(\hat{R}_n \cos\theta_{\lambda,n}^{(1)})^2 | \lambda \text{ not sent}\} = \tfrac{1}{2}L \tag{2.42}$$

Hence,

$$\mathrm{Var}(\hat{R}_n \cos\theta_{\lambda,n}^{(1)} | \lambda \text{ sent}) = (\tfrac{1}{2}L) - (\tfrac{1}{4}L\varepsilon_\Gamma)^2 \tag{2.43}$$

$$\approx \tfrac{1}{2}L$$

because of our small signal approximation.

$$\mathrm{Var}(\hat{R}_n \cos\theta_{\lambda,n}^{(1)} | \lambda \text{ not sent}) = \tfrac{1}{2}L \tag{2.44}$$

The moments of $\mathcal{R}_\lambda$ and $\mathcal{R}_\mu$ are now given by

$$E\{\mathcal{R}_\lambda | \lambda \text{ sent}\} = -\tfrac{1}{4}NL\varepsilon_\Gamma \tag{2.45}$$

$$E\{\mathcal{R}_\mu | \lambda \text{ sent}\} = 0 \tag{2.46}$$

$$\mathrm{Var}\{\mathcal{R}_\lambda | \lambda \text{ sent}\} \approx \mathrm{Var}\{\mathcal{R}_\mu | \lambda \text{ sent}\} = NL/2 \tag{2.47}$$

Consequently, the signal to noise ratio is

$$\mathrm{SNR} = (NL)^{\frac{1}{2}} \tfrac{1}{4}\varepsilon_\Gamma \tag{2.48}$$

If the interfering noise process n(t) was a Gaussian process, then the samples $\eta_n$ would have Gaussian distribution and the solution of Equation (2.36) would yield

$$L = (\mathrm{Var}(\eta_n))^{-1} = \sigma_G^{-2} \tag{2.49}$$

Additionally, the receiver would be completely linear, because

$$\hat{R}_n = -R_n \tag{2.50}$$

Finally, the signal to noise ratio would become

$$\mathrm{SNR} = (N)^{\frac{1}{2}} \tfrac{1}{4}\varepsilon_{\Gamma}/\sigma_G \tag{2.51}$$

Equation (2.51) causes us to define the processing gain of the zero memory nonlinearity as follows

$$\mathrm{ZNLPG} = L\ \mathrm{var}(\eta_n) \tag{2.52}$$

Since our input noise has unbounded variance the ZNL "processing gain" is also unbounded. Clearly a ZNL of some sort will always be required in CDMA systems suffering from impulsive noise. In the next section, we will calculate L for our noise model.

## 2.5 Performance in Combined Atmospheric Noise and Multiple Access Interference

In this section, we will calculate L as a function of the parameters in our noise model. From Equation (2.39), L is

$$L = \int_{-\infty}^{\infty} \frac{\left(\frac{d}{dx} p_{\eta_n}(x)\right)^2}{p_{\eta_n}(x)} dx \tag{2.53}$$

where $\eta_n$ is the $n^{th}$ noise sample. This expression is difficult to evaluate directly, so we integrate by parts and take limits to obtain

$$L = -\int_{-\infty}^{\infty} \log p_{\eta_n}(x) \frac{d^2}{dx^2} p_{\eta_n}(x)\, dx \tag{2.54}$$

Symmetry considerations provide

$$L = -2\int_{0}^{\infty} \log p_{\eta_n}(x) \frac{d^2}{dx^2} p_{\eta_n}(x) dx \tag{2.55}$$

The characteristic function of the random variable $\eta_n$ may be defined as the Fourier transform of the first order probability distribution function and is denoted $\phi_\eta(\omega)$. Characteristic functions are one way to obtain an expression for L that can be readily evaluated via computer. Consequently, we observe that

$$p_{\eta_n}(x) = (1/\pi) \int_{0}^{\infty} \phi_\eta(\omega) \cos\omega x \, d\omega \tag{2.56}$$

$$\frac{d^2}{dx^2} p_{\eta_n}(x) = (-1/\pi) \int_{0}^{\infty} \phi_\eta(\omega)\omega^2 \cos\omega x \, d\omega \tag{2.57}$$

Each noise sample is a sum as follows

$$\eta_n = \eta_C + \eta_G \sum_{\beta=2}^{K} \eta^{(\beta)} \tag{2.58}$$

where $\eta^{(\beta)}$ is the portion of the noise due to the $\beta^{th}$ competing user. Since each term on the right hand side of Equation (2.58) is independent, the characteristic function $\phi_\eta(\omega)$ is a product

$$\phi_\eta(\omega)=\phi_C(\omega)\phi_G(\omega)(\phi^{(\beta)}(\omega))^{K-1} \tag{2.59}$$

If the individual characteristic functions are known, the ZNL processing gain may be computed.

As discussed in Section 2.3, the pdf of $\eta_C$ is being modelled as a Cauchy pdf. Consequently,

$$\phi_C(\omega) = \exp(-|\omega A_1/2|) \tag{2.60}$$

The Gaussian portion of the noise has pdf

$$p_{\eta_G}(z) = \frac{1}{\sqrt{2\pi}\sigma_G} \exp(-z^2/2\sigma_G^2) \tag{2.61}$$

and characteristic function

$$\phi_G(\omega) = \exp(-(\omega\sigma_G)^2/2) \tag{2.62}$$

Finally, the multiple access interference introduced by each competing user is

$$\eta^{(\beta)} = \tfrac{1}{2}\varepsilon_{\Gamma} \cos\theta^{(\beta)} \tag{2.63}$$

where $\theta^{(\beta)}$ may be modelled as a uniform random variable on $[0,2\pi)$ as discussed in Section 3.3. Each $\eta^{(\beta)}$ has the following distribution

$$p_{\eta^{(\beta)}}(z) = \begin{cases} \dfrac{2}{\varepsilon_{\Gamma}\pi} \left(1-(2z/\varepsilon_{\Gamma})^2\right)^{-\frac{1}{2}} & -\frac{1}{2}\varepsilon_{\Gamma} \leq z \leq \frac{1}{2}\varepsilon_{\Gamma} \\ 0 & \text{otherwise} \end{cases} \tag{2.64}$$

The variance of this noise

$$(\sigma^{(\beta)})^2 = \frac{\varepsilon_{\Gamma}^2}{8} \tag{2.65}$$

and characteristic function is

$$\phi^{(\beta)}(\omega) = J_0(\omega\sqrt{2}\sigma^{(\beta)}) \tag{2.66}$$

where $J_0$ is the Bessel function of order zero.

The above equations have been used to plot a normalized Fisher's information number defined as follows

$$\hat{L} = (\mathrm{var}(\eta_G) + (K-1)\mathrm{var}(\eta^{(\beta)}))\ L \tag{2.67}$$

In Figures 2.4 through 2.8, L is plotted versus the variance of the Gaussian noise component and with the number of users (K) as a parameter. Figure 2.4 is for $A_1$=.01 and var($\eta^{(\beta)}$)=.001. Figures 2.5, 2.6, 2.7, and 2.8 are for $A_1$=.1 and var($\eta^{(\beta)}$)=0.001, 0.01, 0.1, and 1.0 respectively.

The above equations have also been used to plot the form of the ZNL given by

$$\hat{R}_n = -\frac{d}{dR_n} \log p_{\eta_n}(R_n) \qquad (2.68)$$

In Figures 2.9 and 2.10, the ZNL is plotted for K=11, variance($\eta_G$)=.1, and $A_1$=.01 and $A_1$=.1 respectively.

We now discuss the figures. The most important feature of Figures 2.4 through 2.8 is that L approaches 1 with increasing K or Gaussian noise variance. In other words, as multiple access noise power <u>or</u> Gaussian noise power increases, Fisher's information number approaches their combined variance. This means that the LOBD performance is close to the performance of a linear receiver without the impulsive noise. To see why, consider the ZNLs plotted in Figures 2.9 and 2.10. All these optimal ZNLs are linear for small signals and strongly suppress signals above a certain threshold. The threshold value is approximately 3.5 times the standard deviation of the combined multiple access and Gaussian noise. As the multiple access or Gaussian noise power increases, the receiver spends more of its time operating in the linear region. At the same time, the impulsive events that do occur are suppressed by the

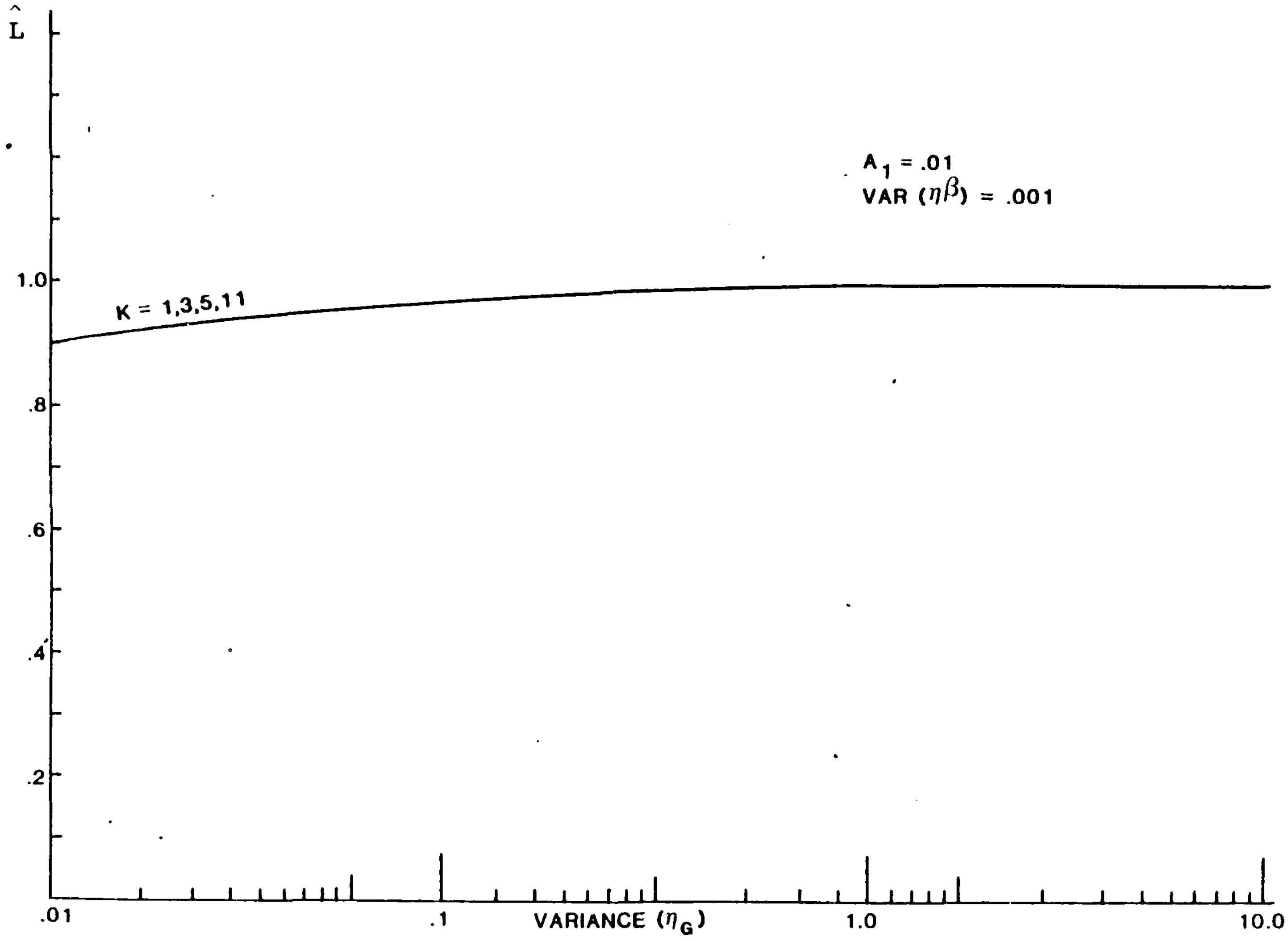


Figure 2.4 Normalized Fisher's Information Number versus Variance of Gaussian Noise Component

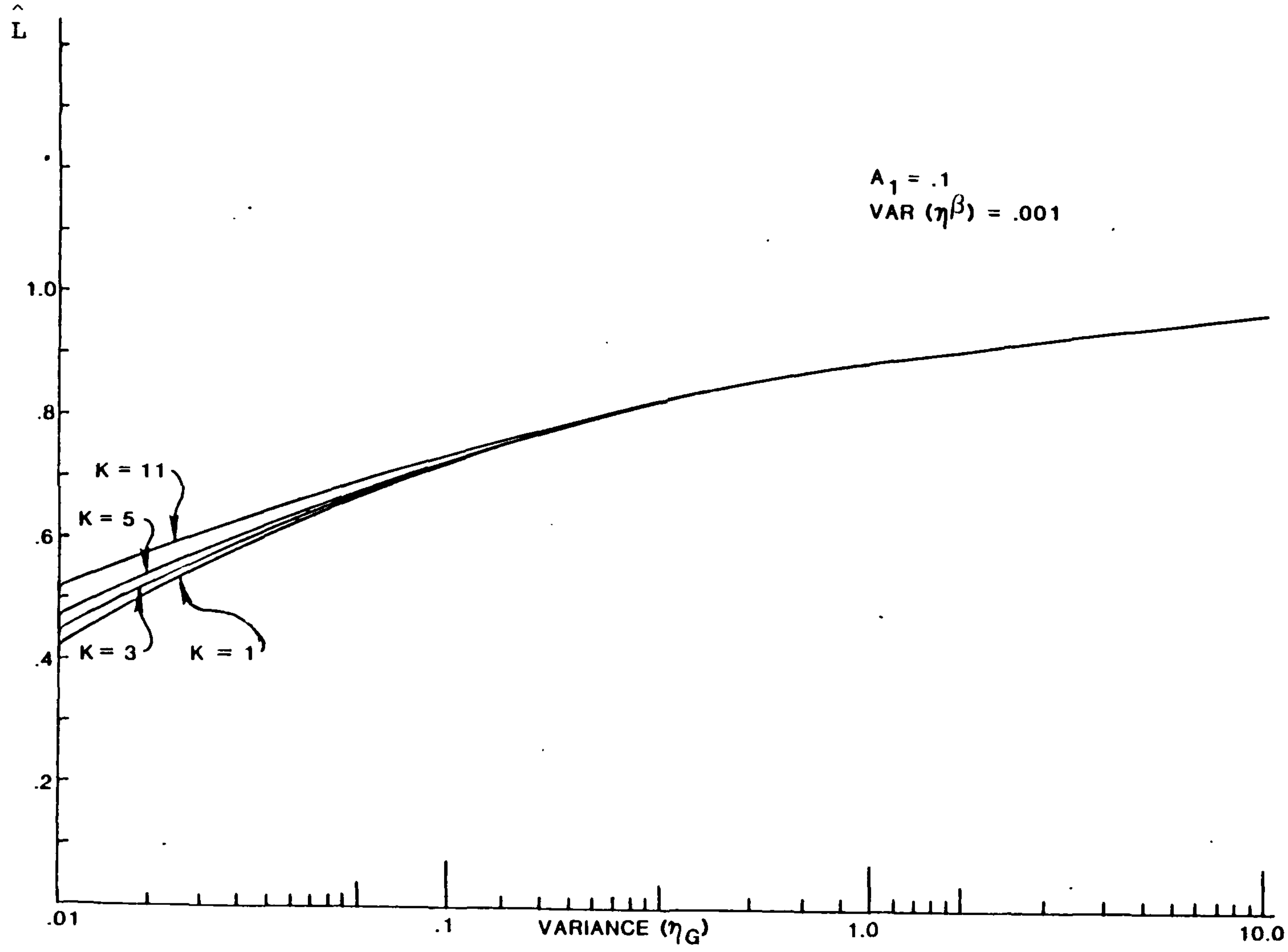


Figure 2.5 Normalized Fisher's Information Number versus Variance of Gaussian Noise Component

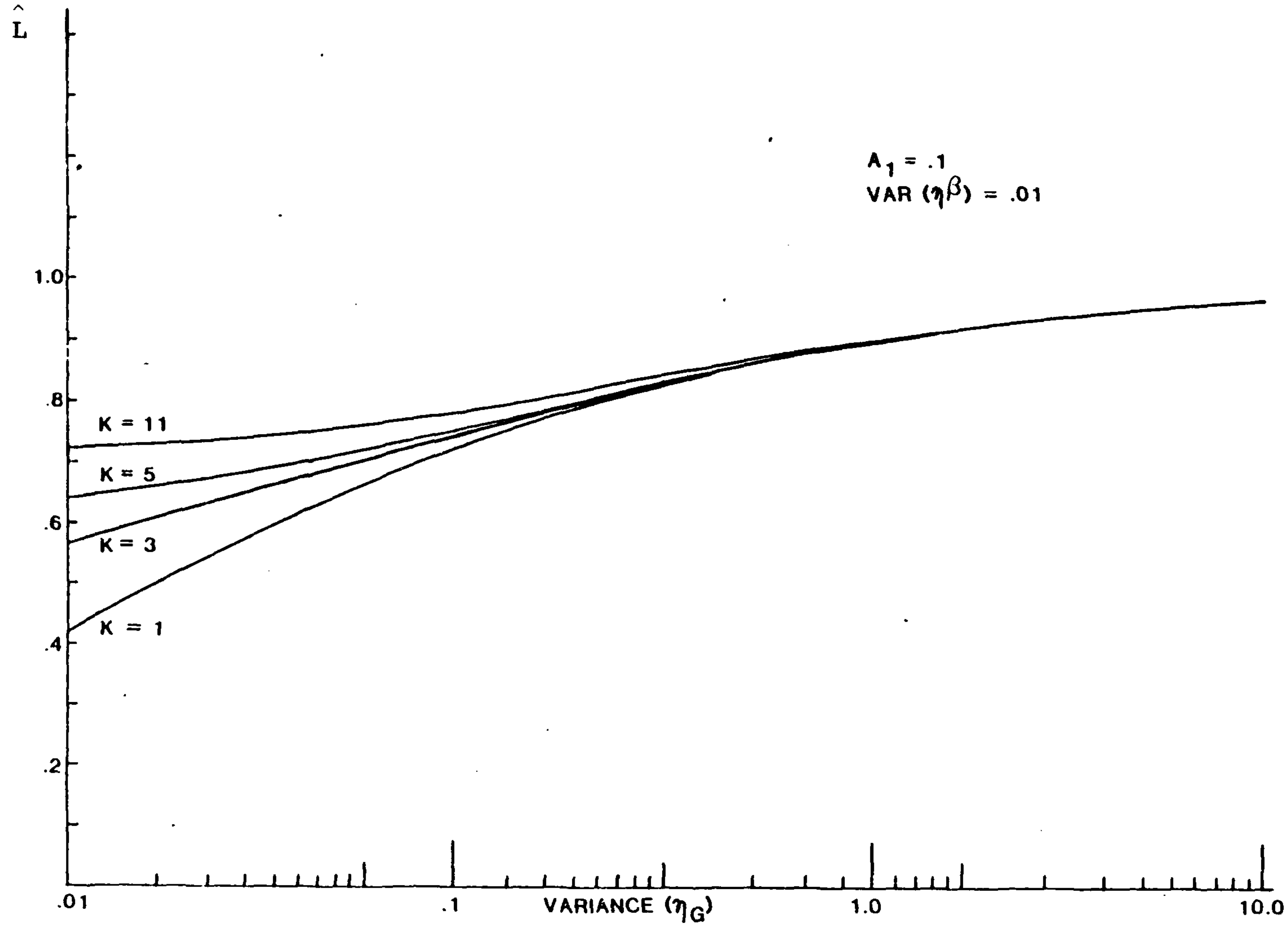


Figure 2.6 Normalized Fisher's Information Number versus Variance of Gaussian Noise Component

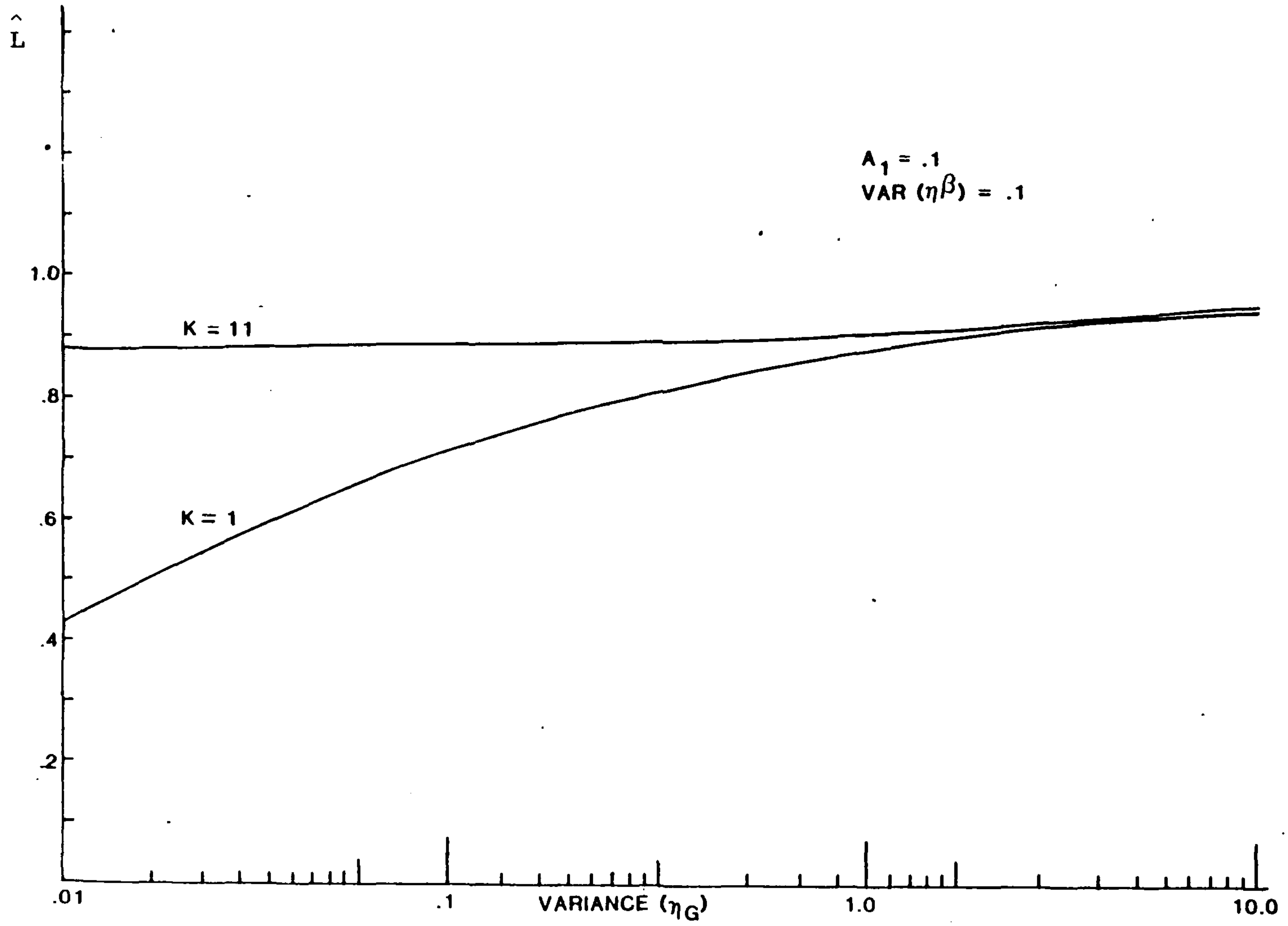


Figure 2.7 Normalized Fisher's Information Number versus Variance of Gaussian Noise Component

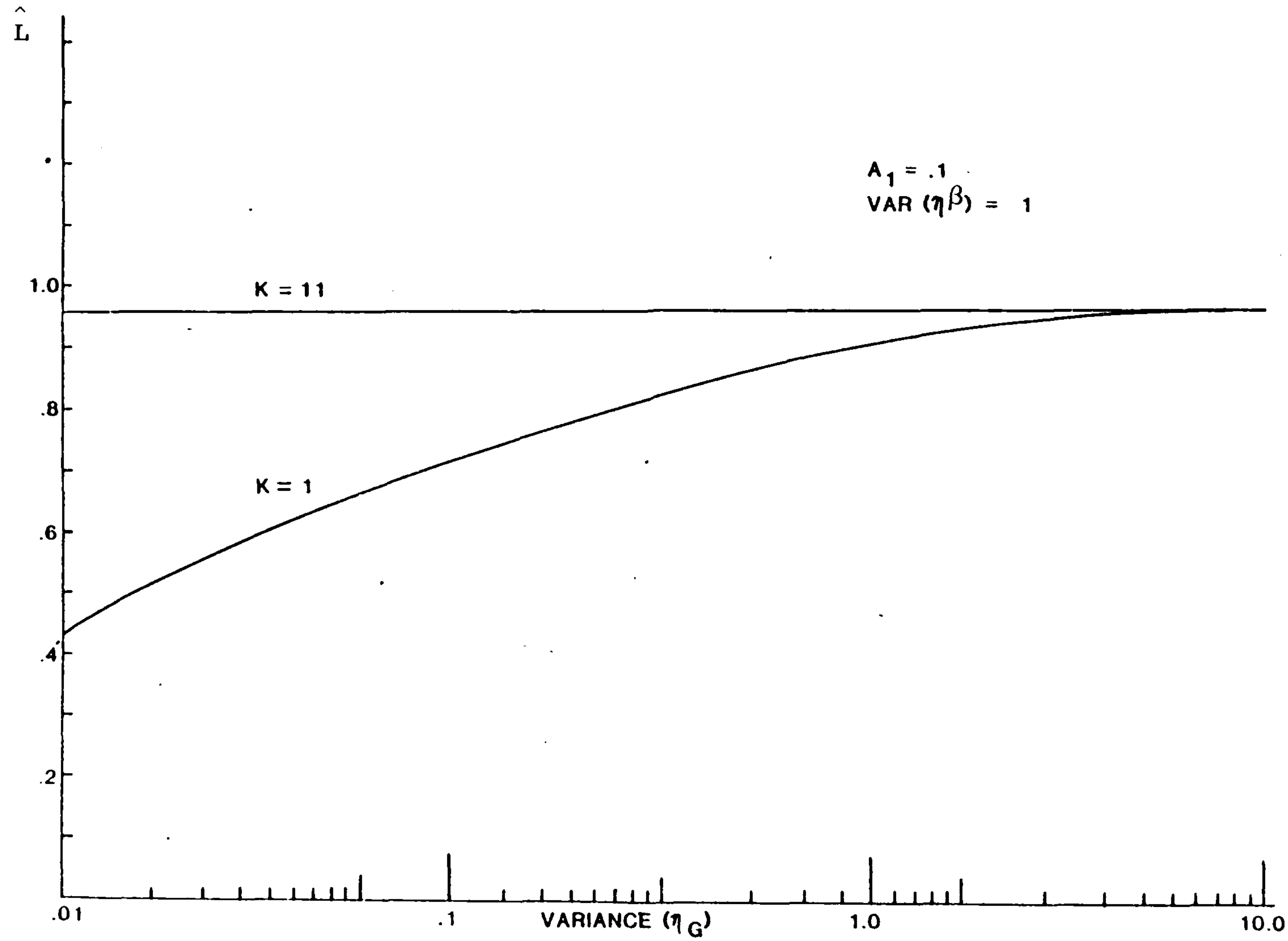


Figure 2.8 Normalized Fisher's Information Number versus Variance of Gaussian Noise Component

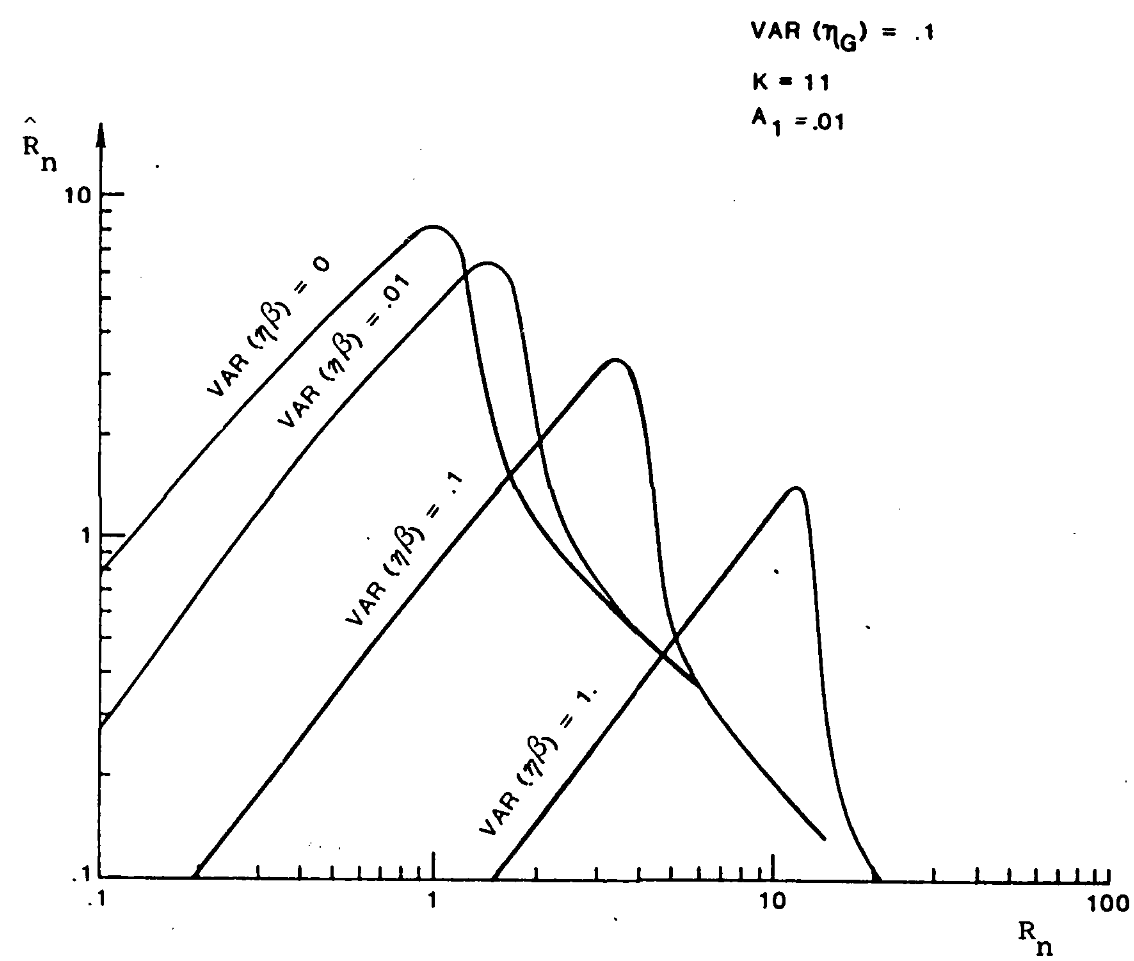


Figure 2.9 Optimal ZNL for Combined Cauchy, Gaussian, and Multiple Access Noise

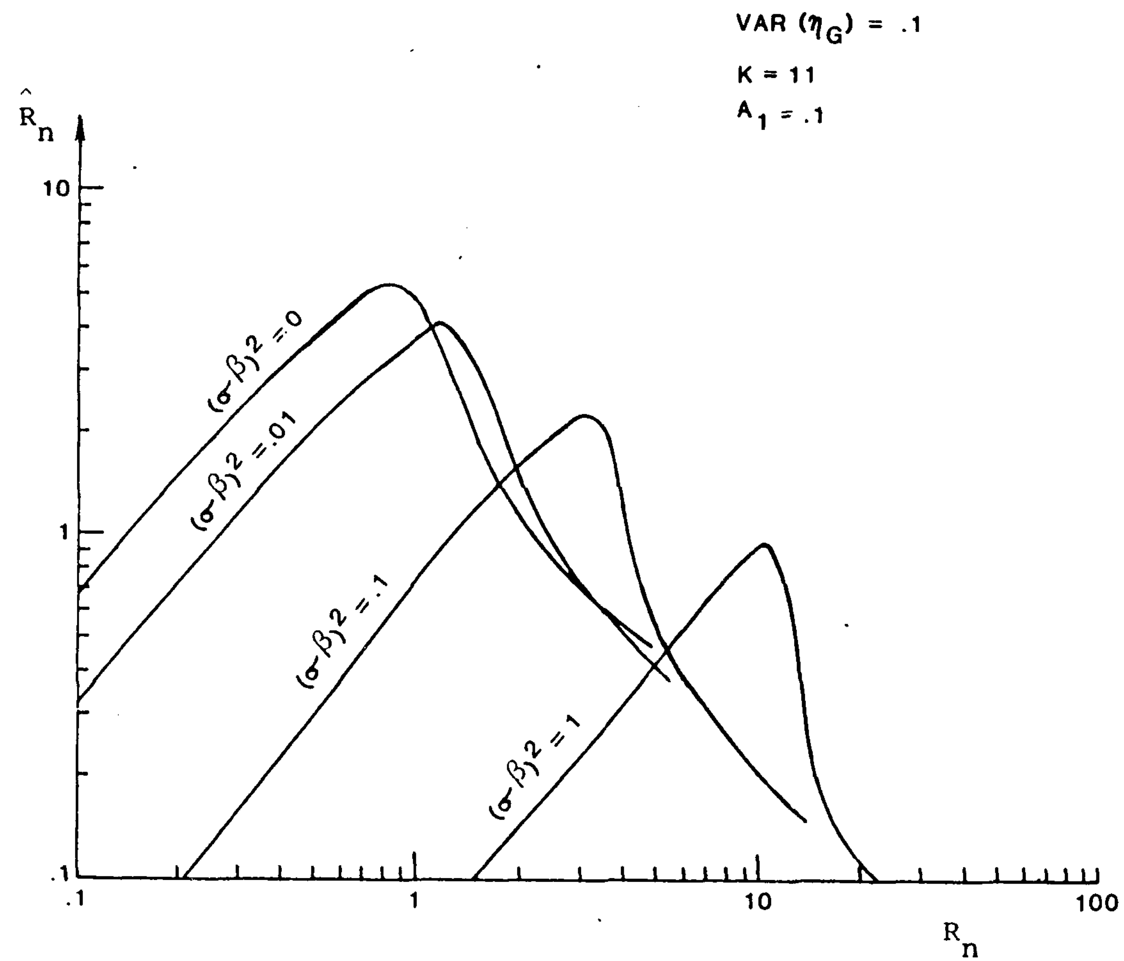


Figure 2.10 Optimal ZNL for Combined Cauchy, Gaussian, and Multiple Access Noise

nonlinearity. From the above observations, we draw two major conclusions.

First of all, every LF/CDMA receiver should employ some form of nonlinearity to suppress the effect of the impulsive noise. This nonlinearity need not be as complicated as the ones given in Figures 2.9 or 2.10. Certainly, a simple clipper or hole puncher will work nearly as well. These simpler ZNLs could be designed to suppress inputs greater than 3.5 times the standard deviation of the combined multiple access and Gaussian noise. Alternatively, the threshold could be designed to adapt such that a certain percentage of the input samples where suppressed (Feldman (1972)).

Our second major conclusion is that the LOBD operating in combined multiple access, Gaussian and impulsive noise has performance close to a linear receiver operating in the same environment <u>without</u> the impulsive noise. The Gaussian noise power in the linear analysis could be adjusted to include the integrated effect of a large number of clipped impulsive events.

# CHAPTER 3

# SYSTEM ANALYSIS PART II:

# LINEAR RECEIVERS

## 3.1 Introduction

In this chapter, we consider linear receivers operating in the presence of additive white Gaussian noise (AWGN). In the first section, we will only consider a single user system, but in Section 3.4, we will consider multiple users. Between Sections 3.2 and 3.4, we will discuss the random sequence analysis ideas, which are important to our multiuser analysis.

## 3.2 Single User Systems

Our linear receiver is shown in Figure 3.1 and it simply consists of M correlators and a "largest of" decision device. Each correlator is matched to one of the signals from the code and if K equals one

$$\mathcal{R}_\lambda = \begin{cases} \dfrac{N\varepsilon_\Gamma}{2} + I_{A,\lambda} & \text{if } \lambda \text{ sent} \\ \\ I_{A,\lambda} & \text{if } \lambda \text{ not sent} \end{cases} \tag{3.1}$$

$I_{A,\lambda}$ is the inner product of $s(v^{(1)},t)$ and the AWGN process, which has power spectral density equal to $N_o/2$.

The performance of our linear receiver (in Figure 3.1) for single users and AWGN is analyzed in many communication texts such as VanTrees (1968) and Wozencraft (1965). The union bound can be used to provide the following well-known probability of

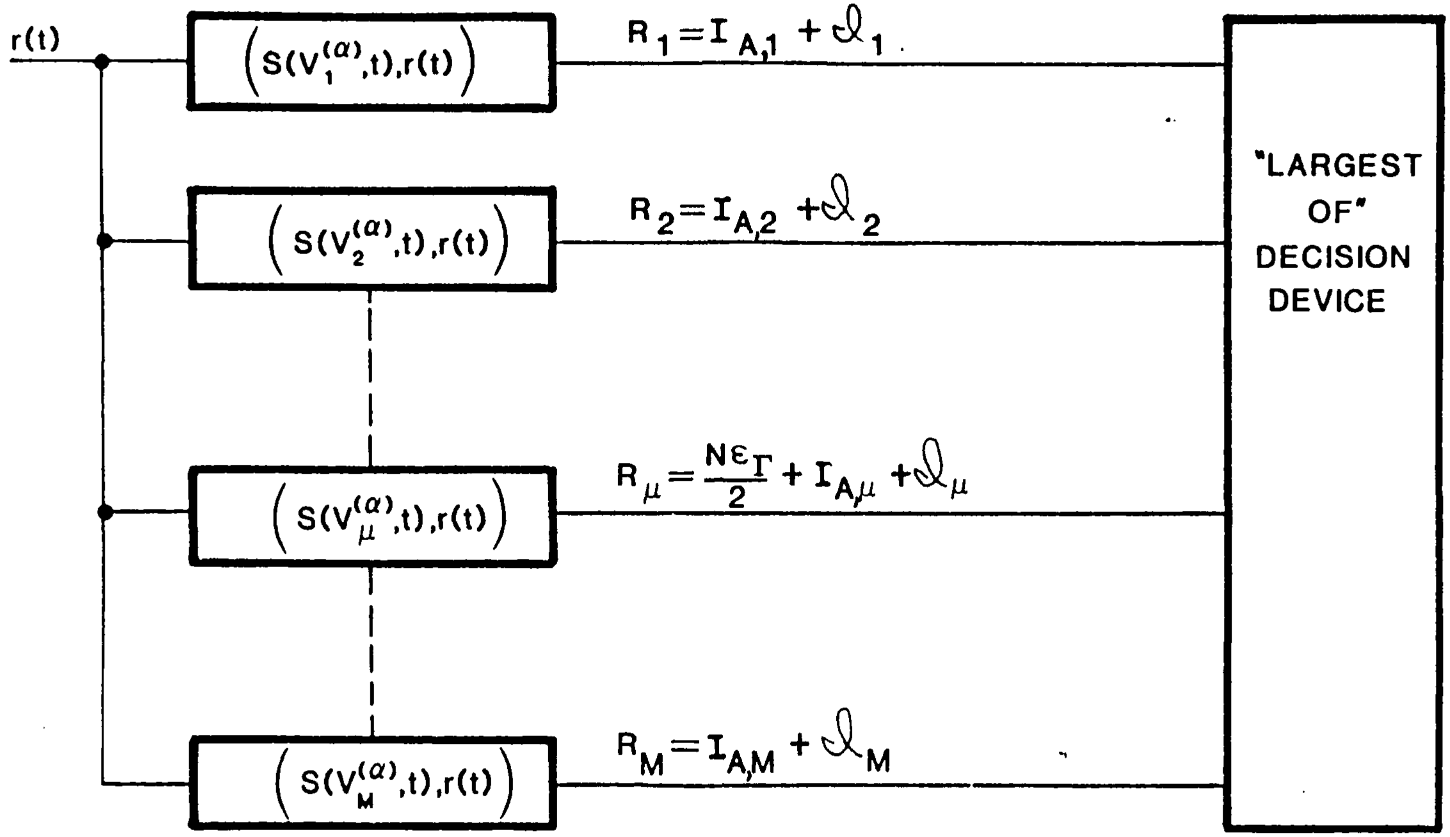


Figure 3.1 Linear Receiver

codeword error bounds

$$Q(\sqrt{\varepsilon_s/N_o}) \leq Pr(\varepsilon) \leq (M-1)Q(\sqrt{\varepsilon_s/N_o}) \quad (3.2)$$

where the codewords are orthogonal and Q is given in Equation (2.41). For comparing sequence sets of different sizes, a bound for $Pr(\varepsilon)$ as a function of the energy per information bit $\varepsilon_b$ is more useful. Such an expression is made possible by using

$$\varepsilon_b = \varepsilon_s / \log_2 M \quad (3.3)$$

Now,

$$Q(\sqrt{\varepsilon_b \log_2 M/N_o}) \leq Pr(\varepsilon) \leq (M-1)Q(\sqrt{\varepsilon_b \log_2 M/N_o}) \quad (3.4)$$

Throughout this thesis, we will assume each user's code is orthogonal, and we will now briefly describe a construction for these codes, which is suitable for a single user system. An orthogonal code may be constructed as follows

$$C^{(O)} = \{v_\lambda\}_{\lambda=1}^{M} \quad (3.5)$$

$$v_\lambda = (v_{\lambda,0} \cdots v_{\lambda,N-1}) \quad (3.6)$$

$$v_{\lambda,n} = \exp(j2\pi\lambda n/N) \quad (3.7)$$

This code may be represented by a matrix D, whose rows are the code words

$$D=\begin{bmatrix} v_{0,0} & v_{0,1} & \cdots & v_{0,N-1} \\ \vdots & & & \\ v_{N-1,0} & \cdots & & v_{N-1,N-1} \end{bmatrix} \tag{3.8}$$

The matrix D is the discrete Fourier transform matrix and the corresponding orthogonal codes have the following properties

$$r=N \qquad M=N \tag{3.9}$$

Orthogonal codes (constructed in any way) are characterized by their probability of error bound versus $\varepsilon_b/N_o$ ratio in Figure 3.2, where sequence length (or equivalently, code size) is a parameter.

## 3.3 Random Sequence Analysis

In the last section, we analyzed the performance of our linear receiver for the case of one user, and in the next section, we will expand our analysis to the multiuser case. For part of our multiuser analysis, we will use random sequence analysis. Hence, we use this section to continue the discussion of random sequence analysis, which we began in Section 1.2. As mentioned, taking the expected value of system performance parameters with respect to the random sequences makes it possible to get useful approximate results without designing any actual

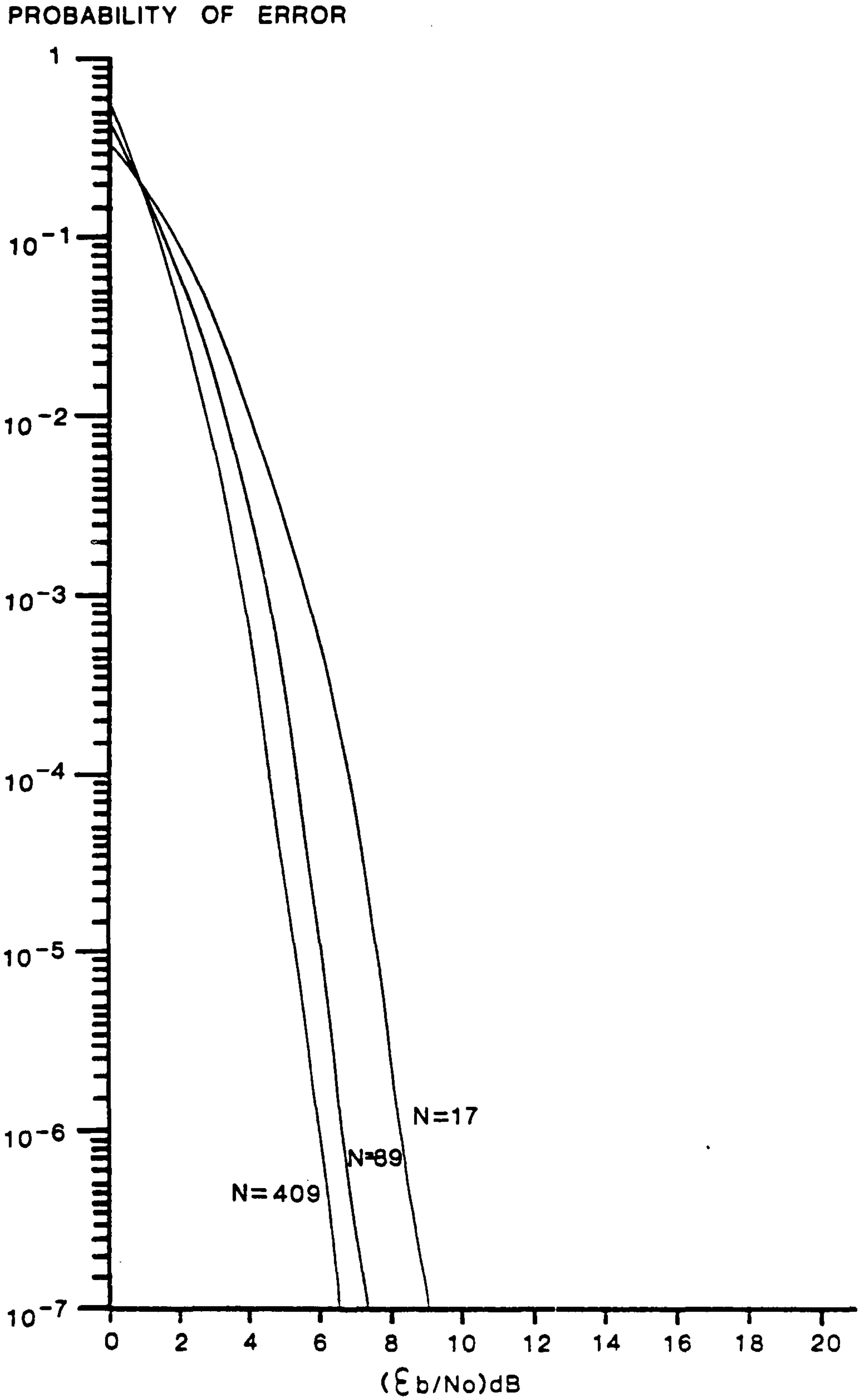


Figure 3.2 Single User Probability of Error

codes. The role of random analysis in the analysis of CDMA communication systems is explained in Roefs and Pursley (1976) and Pursley (1977).

For some of our multiuser analysis, each user's code will be treated as a random coset of an orthogonal code. In other words, each codeword in $C^{(\alpha)}$ consists of the element by element product of a codeword from an orthogonal code and a random vector (or coset leader). Mathematically, if

$$C^{(0)}=\{x_{\lambda}^{(0)}\}_{\lambda=1}^{M} \tag{3.10}$$

is an orthogonal code, then every codeword in $C^{(\alpha)}$ may be expressed as

$$v_{\lambda}^{(\alpha)}=(x_{\lambda,0}^{(0)}\cdot c_{0}^{(\alpha)},\ x_{\lambda,1}^{(0)}\cdot c_{1}^{(\alpha)},\ \ldots\ ,x_{\lambda,N-1}^{(0)}\cdot c_{N-1}^{(\alpha)}) \tag{3.11}$$

where the $c_n^{(\alpha)}$ are independent identically distributed random variables. Additionally,

$$c_n^{(\alpha)}=\exp j\theta_{c,n}^{(\alpha)} \tag{3.12}$$

For our work, the $\theta_{c,n}^{(\alpha)}$ will be uniformly distributed from 0 to $2\pi$, because r is so large that a uniform distribution is a good approximation. Expectation with respect to the random vector $c^{(\alpha)}$ will be denoted $E_c^{(\alpha)}$.

Random sequence analysis will be used in the next section to approximate the probability of error for our LF/CDMA model as a function of the number of users.

## 3.4 Multiuser Analysis

### 3.4.1 Introduction

In this subsection, we develop a probability of error estimate and bound for the multiuser system model. The estimate is based on the assumption that the interference introduced by the multiple users has a Gaussian distribution. Consequently, it only requires the calculation of the variance of the multiple access interference. Second, we bound the system probability of error via the characteristic function method. This technique is pioneered for CDMA systems in Geraniotis and Pursley (1982) and Geraniotis (1983) and we obtain some interesting extensions.

For our multiuser analysis, we will continue to use the receiver of Figure 3.1, but now the output of each correlator is given by

$$\mathcal{R}_\lambda = \begin{cases} \dfrac{N\varepsilon_\Gamma}{2} + I_{A,\lambda} + \mathcal{I}_\lambda & \lambda \text{ sent} \\ \\ I_{A,\lambda} + \mathcal{I}_\lambda & \lambda \text{ not sent} \end{cases} \tag{3.13}$$

where $\mathcal{I}_\lambda$ is the multiple access interference. The multiple access interference $\mathcal{I}_\lambda$ is the sum of interference introduced by each of the competing users:

$$\mathcal{I}_\lambda = \sum_{\beta=2}^{K} I_\lambda^{(\beta)} \tag{3.14}$$

where

$$I_{\lambda}^{(\beta)} = \int_0^{NT_c} s(v_{\lambda}^{(1)},t)\,s(b^{(\beta)},t+\tau^{(\beta)})\,dt \tag{3.15}$$

To rewrite Equation 3.15 and to make analysis simpler we introduce some new functions. First of all, the correlation between the signature sequence x and the concatenation of the sequences u and v is

$$H_{x,u,v}(\ell) = \sum_{n=\ell}^{N-1} x^*_{n-\ell} u_n + \sum_{n=0}^{\ell-1} x^*_{n+N-\ell} v_n \tag{3.16}$$

This function may be related to the aperiodic cross-correlation functions described in Sarwate and Pursley (1980):

$$H_{x,u,v}(\ell) = C^*_{x,u}(\ell) + C_{v,x}(N-\ell) \tag{3.17}$$

Also note that

$$H_{x,u,u}(\ell) = \theta_{x,u}(\ell) \tag{3.18}$$

where $\theta_{x,u}(\ell)$ is the periodic cross-correlation function which is also extensively discussed in Sarwate and Pursley (1980). In addition, the partial autocorrelation functions of the chip waveform ($\Gamma(t)$) are given by (Pursley, 1982)

$$\hat{R}_{\Gamma}(s) = \int_s^{T_c} \Gamma(t)\,\Gamma(t-s)\,dt \qquad 0 \le s \le T_c \tag{3.19}$$

and

$$R_{\Gamma}(s) = \int_0^{s} \Gamma(t)\,\Gamma(t+T_c-s)\,dt \qquad 0 \le s \le T_c \tag{3.20}$$

For our preferred example chip waveform (the sine pulse), these functions may be developed to yield (Pursley, 1982)

$$\hat{R}_\Gamma(s)=\frac{\varepsilon_\Gamma}{T_c}\left((T_c-s)\cos(\pi s/T_c)+(T_c/\pi)\sin(\pi s/T_c)\right) \tag{3.21}$$

$$R_\Gamma(s)=\frac{\varepsilon_\Gamma}{T_c}\left(-s\cos(\pi s/T_c)+(T_c/\pi)\sin(\pi s/T_c)\right) \tag{3.22}$$

Note that

$$\hat{R}_\Gamma(0)=R_\Gamma(T_c)=\varepsilon_\Gamma \tag{3.23}$$

$$\hat{R}_\Gamma(T_c)=R_\Gamma(0)=0 \tag{3.24}$$

With the correlation functions defined above and equation 2.15, we may develop Equation 3.15 as follows

$$\begin{aligned} I_\lambda^{(\beta)}=&\tfrac{1}{2}\hat{R}(\tau^{(\beta)})(\cos\omega_c\tau^{(\beta)}\ \mathrm{Re}\{H_{x,u,v}(\ell)\} \\ &-\sin\omega_c\tau^{(\beta)}\ \mathrm{Im}\{H_{x,u,v}(\ell)\}) \\ &+\tfrac{1}{2}R(\tau^{(\beta)})(\cos\omega_c\tau^{(\beta)}\ \mathrm{Re}\{H_{x,u,v}(\ell+1)\} \\ &-\sin\omega_c\tau^{(\beta)}\ \mathrm{Im}\{H_{x,u,v}(\ell+1)\}) \end{aligned} \tag{3.25}$$

or

$$I_{\lambda}^{(\beta)}=\tfrac{1}{2}\hat{R}(\tau^{(\beta)})\ \mathrm{Re}\{\exp j\omega_c\tau^{(\beta)}(C^*_{x,u}(\ell)+C_{v,x}(N-\ell))\} \tag{3.26}$$

$$+\tfrac{1}{2}R(\tau^{(\beta)})\ \mathrm{Re}\{\exp j\omega_c\tau^{(\beta)}(C^*_{x,u}(\ell+1)+C_{v,x}(N-\ell-1))\}$$

where

$$b^{(\beta)}=(u^{(\beta)}|v^{(\beta)}) \tag{3.27}$$

$$\ell T_c\leq\tau^{(\beta)}<(\ell+1)T_c \tag{3.28}$$

As shown above, the interference introduced by each of the competing users is a function of the sequences sent by that user during the integration time $[0,NT_c)$ as well as the relative delay. In this thesis, both the choice of codewords and the relative delay are modelled as random. Specifically, the $u^{(\beta)}$ and $v^{(\beta)}$ are independent and equally likely to be any sequences in $C^{(\beta)}$. Additionally, $u^{(\beta)}$ and $v^{(\beta)}$ are independent of $u^{(\gamma)}$ and $v^{(\gamma)}$ for all $\beta$ not equal to $\gamma$. Expectation with respect to the $\beta^{th}$ transmitter's random choice of data is denoted

$$E_b^{(\beta)}\{G(b^{(\beta)})\}=(1/M^2)\sum_{u\varepsilon C^{(\beta)}}\ \sum_{v\varepsilon C^{(\beta)}}\ G(b^{(\beta)}) \tag{3.29}$$

Expectation with respect to the choice of data by all the competing users is denoted $E_b$ and the corresponding variance is denoted $\mathrm{var}_b$.

The relative signal delay is treated as a random variable, which is uniformly distributed over the interval $[0, NT_c)$. We denote expectation over this random variable as

$$E_{\tau}^{(\beta)}\{G(\tau^{(\beta)})\}=(1/NT_c)\int_0^{NT_c} G(\tau^{(\beta)})d\tau^{(\beta)} \tag{3.30}$$

Expectation, with respect to all the competing user signal delays, is denoted $E_{\mathcal{T}}$ and the corresponding variance is $\mathrm{var}_{\mathcal{T}}$.

The average probability of error for our receiver is

$$\Pr(\varepsilon)=(1/M)\sum_{\lambda=1}^{M} \Pr(\text{error}\,|\,v_\lambda^{(1)} \text{ sent}) \tag{3.31}$$

Employing the union bound yields

$$\Pr(\varepsilon)\leq(1/M)\sum_{\lambda=1}^{M}\sum_{\mu\neq\lambda} \Pr(\mathcal{R}_\mu>\mathcal{R}_\lambda\,|\,v_\lambda^{(1)} \text{ sent}) \tag{3.32}$$

The union bound upper bounds the probability of the union of some number of events by ignoring the probability of the various joint events. Consequently, it is a tight bound when the sum of probabilities is small (less than $10^{-3}$) and it is not so tight when the probability of error is large. Substituting expressions for $\mathcal{R}_\mu$ and $\mathcal{R}_\lambda$ yields

$$\Pr(\varepsilon)\leq(1/M)\sum_{\lambda=1}^{M}\sum_{\mu\neq\lambda} \Pr(I_{A,\mu}-I_{A,\lambda}+\mathcal{J}_\mu-\mathcal{J}_\lambda>\frac{N\varepsilon_\Gamma}{2}\,|\,\lambda \text{ sent}) \tag{3.33}$$

$$=(1/M)\sum_{\lambda=1}^{M}\sum_{\mu\neq\lambda} \Pr(\frac{2}{N\varepsilon_\Gamma}(I_{A,\mu}-I_{A,\lambda}+\mathcal{J}_\mu-\mathcal{J}_\lambda)>1\,|\,\lambda \text{ sent}) \tag{3.34}$$

The right hand side of this inequality is developed by two different techniques in the next two subsections.

### 3.4.2 Gaussian approximation

The inequality of 3.33 becomes an approximation when the following observations and assumptions are applied. First, observe that $(I_{A,\mu}-I_{A,\lambda})$ is a Gaussian random variable with zero mean and variance $N\varepsilon_{\Gamma}N_o/2$ and it is independent of the multiple access interference. Second, the random variable $(\mathcal{I}_{\mu}-\mathcal{I}_{\lambda})$ has zero mean and its distribution is assumed to be Gaussian, which allows us to write

$$\Pr(\varepsilon)\approx(1/M)\sum_{\lambda=1}^{M}\sum_{\mu\neq\lambda} Q\left[\left[\frac{2N_o}{N\varepsilon_{\Gamma}}+\frac{4}{N^2\varepsilon_{\Gamma}^2}\,\mathrm{var}_{b,\tau}(\mathcal{I}_{\mu}-\mathcal{I}_{\lambda})\right]^{-\frac{1}{2}}\right] \tag{3.35}$$

As shown in Equation (3.22), the multiple access interference contributes to the system performance through its own second moment. Due to the independence of the interference introduced by the different users, we may write

$$\mathrm{var}_{b,\tau}(\mathcal{I}_{\mu}-\mathcal{I}_{\lambda})=\sum_{\beta=2}^{K} E_b^{(\beta)}E_{\tau}^{(\beta)}(I_{\mu}-I_{\lambda})^2 \tag{3.36}$$

If the user codes are orthogonal, we can show that (see Appendix A)

$$E_b^{(\beta)} E_\tau^{(\beta)} (I_\mu - I_\lambda)^2 = (1/8T_c)(\mathcal{M}_\Gamma 4N + \tag{3.37}$$

$$\mathcal{M}_\Gamma' (2\mathrm{Re}\{C_{x_\mu,x_\mu}(1)\} + 2\mathrm{Re}\{C_{x_\lambda,x_\lambda}(1)\} + C^*_{x_\mu,x_\lambda}(-1) + C^*_{x_\mu,x_\lambda}(1)))$$

where

$$\mathcal{M}_\Gamma = \int_0^{T_c} \hat{R}_\Gamma^2(s)\,ds = \int_0^{T_c} R_\Gamma^2(s)\,ds \tag{3.38}$$

and

$$\mathcal{M}_\Gamma' = \int_0^{T_c} R_\Gamma(s)\hat{R}_\Gamma(s)\,ds \tag{3.39}$$

As shown, the variance per interferer depends on the aperiodic cross-correlation and autocorrelation functions of the sequences in $C^{(\alpha)}$ and the chip correlation functions. For our preferred example chip (the sine pulse)

$$\mathcal{M}_\Gamma = \varepsilon_\Gamma^2 T_c (.293) \tag{3.40}$$

$$\mathcal{M}_\Gamma' = \varepsilon_\Gamma^2 T_c (.043) \tag{3.41}$$

This indicates that the variance depends only weakly on the various aperiodic correlation functions and could be approximated as follows.

$$E_b^{(\beta)} E_\tau^{(\beta)} (I_\mu - I_\lambda)^2 \approx \frac{\varepsilon_\Gamma^2 N}{2} (.293) \tag{3.42}$$

Additionally, if our user codes were random cosets of an orthogonal code, we would find

$$E_c^{(\alpha)}\{C_{x_\mu,x_\lambda}(1)\}=0 \tag{3.43}$$

$$E_c^{(\alpha)}\{C^*_{x_\mu,x_\lambda}(\ell)\}=0 \tag{3.44}$$

Hence,

$$E_c^{(\alpha)}E_b^{(\beta)}E_\tau^{(\beta)}(I_\mu-I_\lambda)^2=\frac{N\mathcal{M}_\Gamma}{2T_c} \tag{3.45}$$

We could then write

$$\Pr(\varepsilon)\approx(M-1)Q\left[\left[\frac{2N_o}{N\varepsilon_\Gamma}+\frac{2\mathcal{M}_\Gamma}{NT_c\varepsilon_\Gamma^2}(K-1)\right]^{-\frac{1}{2}}\right] \tag{3.46}$$

For a sine pulse chip waveform

$$\Pr(\varepsilon)\approx(M-1)Q\left[\left[\frac{2N_o}{N\varepsilon_\Gamma}+\frac{.586}{N}(K-1)\right]^{-\frac{1}{2}}\right] \tag{3.47}$$

This result will be plotted for various N and discussed in Subsection 3.4.4, but first we will develop a probability of error bound based on characteristic functions.

### 3.4.3 Characteristic function method

In this subsection, we will use characteristic functions to bound the multiuser error probability (MEP). Characteristic functions have been used to analyze performance in the presence of intersymbol interference and additive Gaussian noise by Shimbo and Celebiler (1971). Additionally, they have been used to analyze the effect of intersymbol interference or Rayleigh fading by Vanelli and Shehadeh (1974). However, they also have been used in the analysis of CDMA systems by Geraniotis and Pursley (1982) and Geraniotis (1983). In Geraniotis and Pursley (1982), binary and quaternary DS/SSMA communications in the presence of additive white Gaussian noise are considered. The characteristic function method is used to give probability of error expressions when specific signature sequences (m-sequences and Gold sequences) are used. In Geraniotis (1983), direct sequence and frequency hopped SSMA communications are studied. In this work, Geraniotis limits his consideration to random sequences, but he includes the effect of fading channels. In what follows, we will draw on both of these results.

In this analysis, we will consider random coset orthogonal codes. The probability of error bound given in Section 3.3.4 may be rewritten

$$\Pr(\varepsilon) \leq (1/M) \sum_{\lambda=1}^{M} \sum_{\mu\neq\lambda} \Pr((2/N\varepsilon_{\Gamma})(I_{A,\mu}-I_{A,\lambda}+\mathcal{N}_{\mu}-\mathcal{N}_{\lambda})>1 \mid \lambda \text{ sent}) \quad (3.48)$$

$$= (1/M) \sum_{\lambda=1}^{M} \sum_{\mu\neq\lambda} (1-F(1)) \quad (3.49)$$

where F is the cumulative distribution function of the normalized noise plus multiple access interference. If $\phi(\omega)$ denotes the characteristic function of the normalized noise plus interference, then

$$F(1)=(1/2\pi)\int_{-\infty}^{\infty}(1/j\omega)\exp(j\omega)\phi(\omega)\,d\omega+\tfrac{1}{2} \tag{3.50}$$

Since the atmospheric noise and multiple access interference terms are all independent, we may write

$$\phi(\omega)=\phi_I(2\omega/N\varepsilon_\Gamma)\prod_{\beta=2}^{K}\phi^{(\beta)}(2\omega/N\varepsilon_\Gamma) \tag{3.51}$$

where $\phi_I(\omega)$ is the characteristic function of $(I_{A,\mu}-I_{A,\lambda})$ and $\phi^{(\beta)}(\omega)$ is the characteristic function of $I_\mu^{(\beta)}-I_\lambda^{(\beta)}$. If Equation (3.51) is rearranged as follows

$$\phi(\omega)=\phi_\eta(2\omega/N\varepsilon_\Gamma)-\phi_\eta(2\omega/N\varepsilon_\Gamma)\Big(1-\prod_{\beta=2}^{K}\phi^{(\beta)}(2\omega/N\varepsilon_\Gamma)\Big) \tag{3.52}$$

and substituted into Equation (3.49), the following result is achieved

$$\Pr(\varepsilon)\le(M-1)Q((\varepsilon_s/N_o)^{\frac{1}{2}}) \tag{3.53}$$

$$+(1/M)\sum_{\lambda=1}^{M}\sum_{\mu\neq\lambda}(1/2\pi)\int_{-\infty}^{\infty}(1/j\omega)\exp(j\omega)\phi_I(2\omega/N\varepsilon_\Gamma)$$

$$\cdot\Big(1-\prod_{\beta=2}^{K}\phi^{(\beta)}(2\omega/N\varepsilon_\Gamma)\Big)d\omega$$

The first term on the right of this inequality results from noise alone while the second term is the increase in the probability of error due to the multiple users.

Since $I_{A,\mu} - I_{A,\lambda}$ has a Gaussian distribution, its characteristic function is given by

$$\phi_I(\omega) = \exp(-\omega^2 \varepsilon_\Gamma N N_o/4) \tag{3.54}$$

Unfortunately, the characteristic function $\phi^{(\beta)}(\omega)$ is not so straightforward, because, after all

$$\phi^{(\beta)}(\omega) = E_c^{(\alpha)} E_c^{(\beta)} E_b^{(\beta)} E_\tau^{(\beta)} \{\exp(-j\omega(I_\mu^{(\beta)} - I_\lambda^{(\beta)})\} \tag{3.55}$$

where the $I_\mu^{(\beta)}$ and $I_\lambda^{(\beta)}$ are given by Equation (3.25). However in Appendix B, we derive

$$\phi^{(\beta)}(\omega) = (1/T_c)\int_0^{T_c} \prod_{n=0}^{N-1} J_o\left(\frac{\omega\hat{R}(\tau)}{2}\sqrt{2}\sqrt{1-\cos(2\pi n/N)}\right) \times \tag{3.56}$$

$$\prod_{n=0}^{N-1} J_o\left(\frac{\omega R(\tau)}{2}\sqrt{2}\sqrt{1-\cos(2\pi n/N)}\right) d\tau$$

where $J_o$ is the Bessel function of order zero

$$J_o(x) = (2/\pi)\int_0^{\pi/2} \cos(x\cos\theta)\, d\theta \tag{3.57}$$

The above expression allows us to write

$$\prod_{\beta=2}^{K} \phi^{(\beta)}(2\omega/N\varepsilon_\Gamma) = \tag{3.58}$$

$$\Big((1/T_c)\int_0^{T_c} \prod_{n=0}^{N-1} J_o\Big(\frac{\omega\hat{R}(\tau)}{N\varepsilon_\Gamma}\sqrt{2}\sqrt{1-\cos(2\pi n/N)}\Big)$$

$$\cdot \prod_{n=0}^{N-1} J_o\Big(\frac{\omega R(\tau)}{N\varepsilon_\Gamma}\sqrt{2}\sqrt{1-\cos(2\pi n/N)}\Big)d\tau\Big)^{K-1}$$

Since this result is independent of $\mu$ or $\lambda$, we get

$$\Pr(\varepsilon) \le (M-1)Q((\varepsilon_s/N_o)^{1/2}) + \tag{3.59}$$

$$(M-1)/\pi\int_0^{\infty}(1/\omega)(\sin\omega)\phi_I(2\omega/N\varepsilon_\Gamma)\Big(1-\prod_{\beta=2}^{K}\phi^{(\beta)}(2\omega/N\varepsilon_\Gamma)\Big)d\omega$$

Expressions (3.58) and (3.59) have been used to generate probability of error curves as a function of $(\varepsilon_b/N_o)$ with K and N as parameters. These curves are presented and discussed in the next section.

### 3.4.4 Numerical results

In Section 3.4.2, we used the Gaussian assumption to approximate the multiuser error probability (MEP) (see expression (3.46)), and in Section 3.4.3, we used characteristic functions to obtain a bound. In this section, the numerical evaluation of the Gaussian and the characteristic function expressions ($P_G$ and

$P_C$ respectively) will be briefly discussed. Then the resultant curves will be presented and discussed. Finally, an alternate characteristic function approach will be proposed and evaluated.

As shown by Expression (3.46), the numerical evaluation of $P_G$ is very straight forward and the computational effort is independent of K or N. On the other hand, the computational effort required for the evaluation of $P_C$ (inequality (3.59)) does not increase with K, but does increase linearly with N.

The expressions $P_C$ and $P_G$ are plotted in Figures 3.3 through 3.7 with the following parameters:

| Figure No. | K | N |
|---|---|---|
| 3.3 | 2,3 | 17 |
| 3.4 | 2,3 | 31 |
| 3.5 | 3,7,11 | 127 |
| 3.6 | 3,7,11 | 257 |
| 3.7 | 3,7,11 | 509 |

All plots are for systems where the chip waveform is a sine pulse and the plots have the energy per bit to noise density ratio as the independent variable.

The characteristic function curves $P_C$ result from the numerical evaluation of Expressions (3.58) and (3.59). As such, they are approximations instead of upper bounds, because of computational errors. However, we believe that they are very close to true upper bounds, because considerable care was taken in the numerical evaluation of (3.58) and (3.59). The numerical

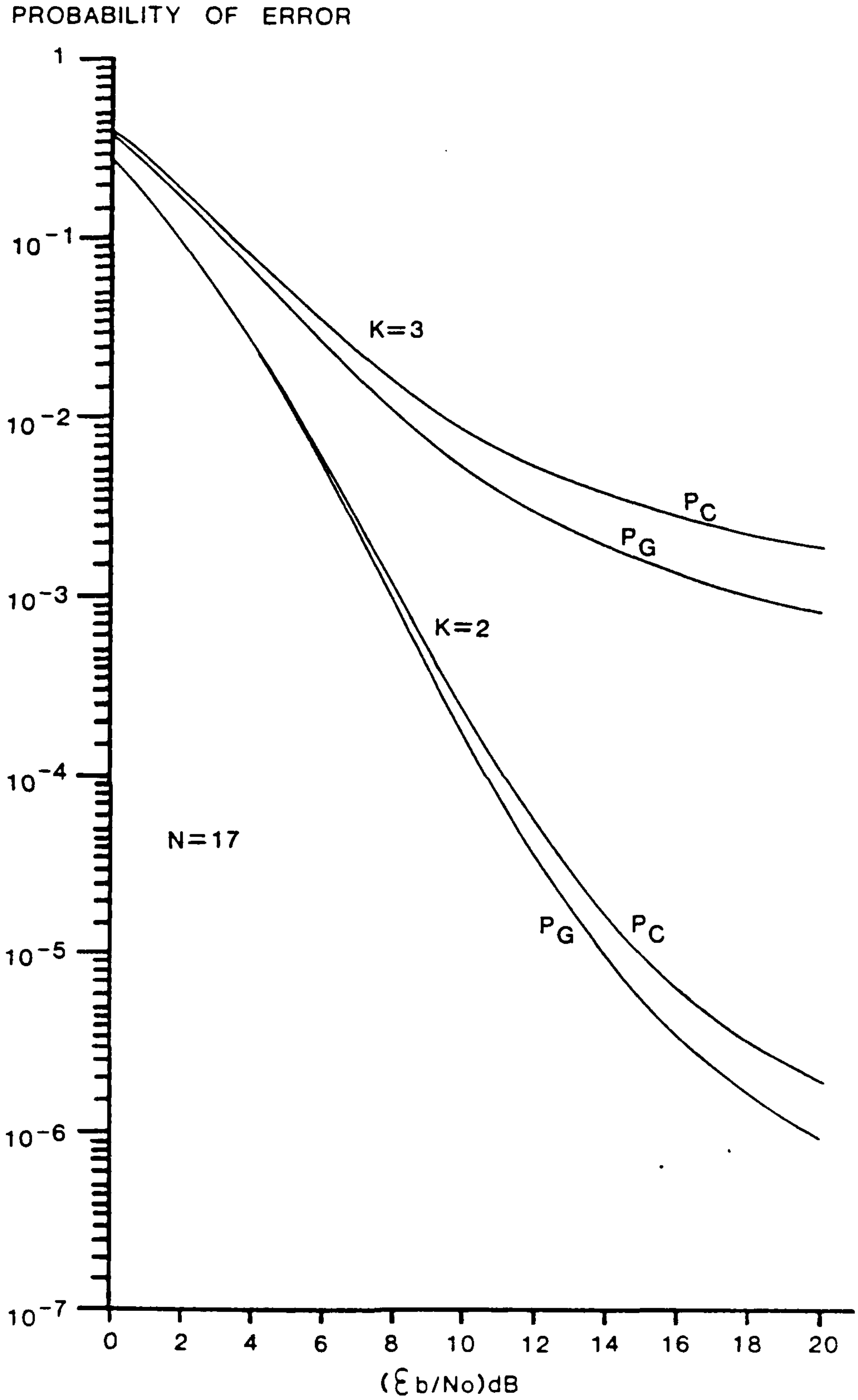


Figure 3.3 N=17 Multiuser Probability of Error

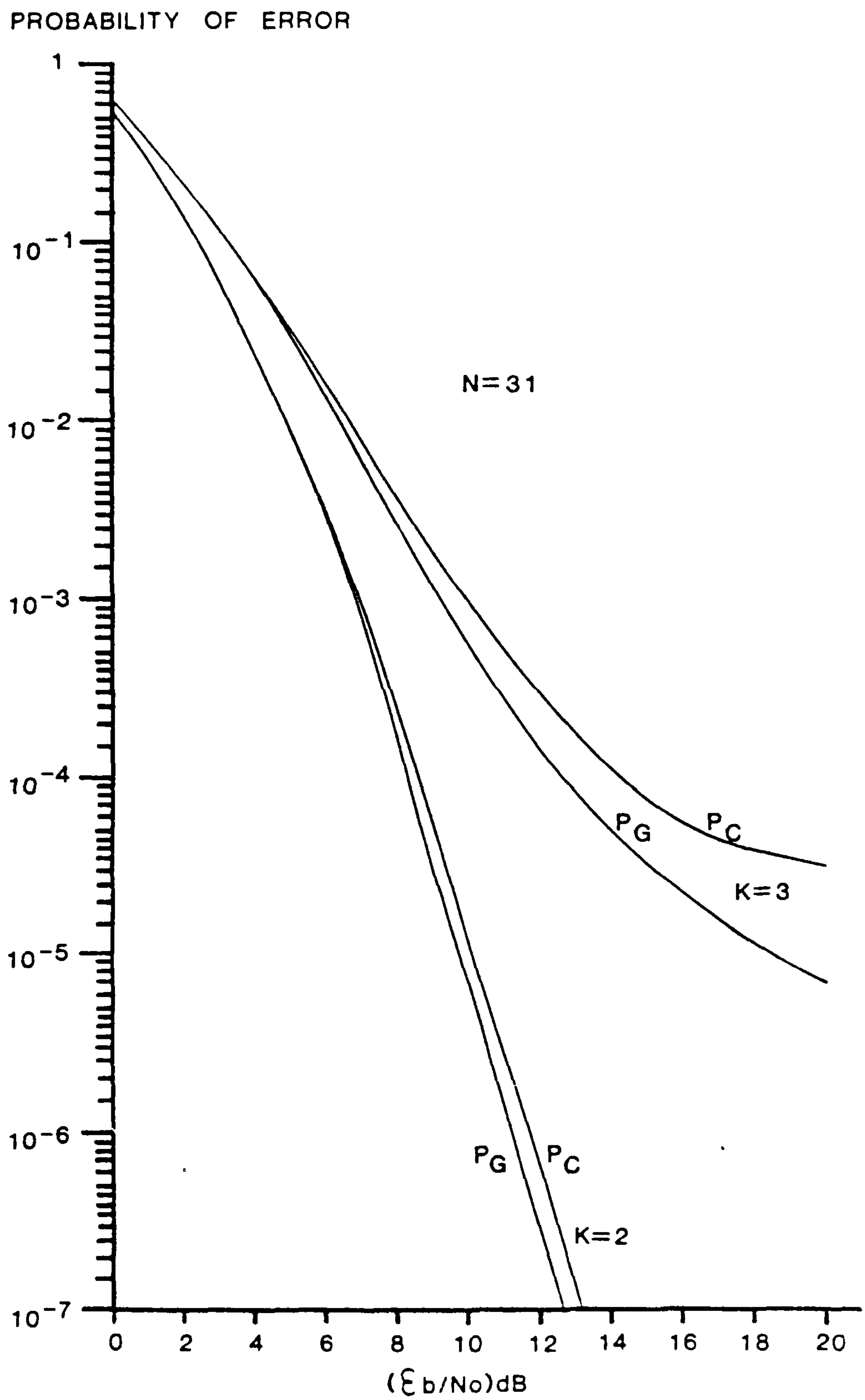


Figure 3.4 N=31 Multiuser Probability of Error

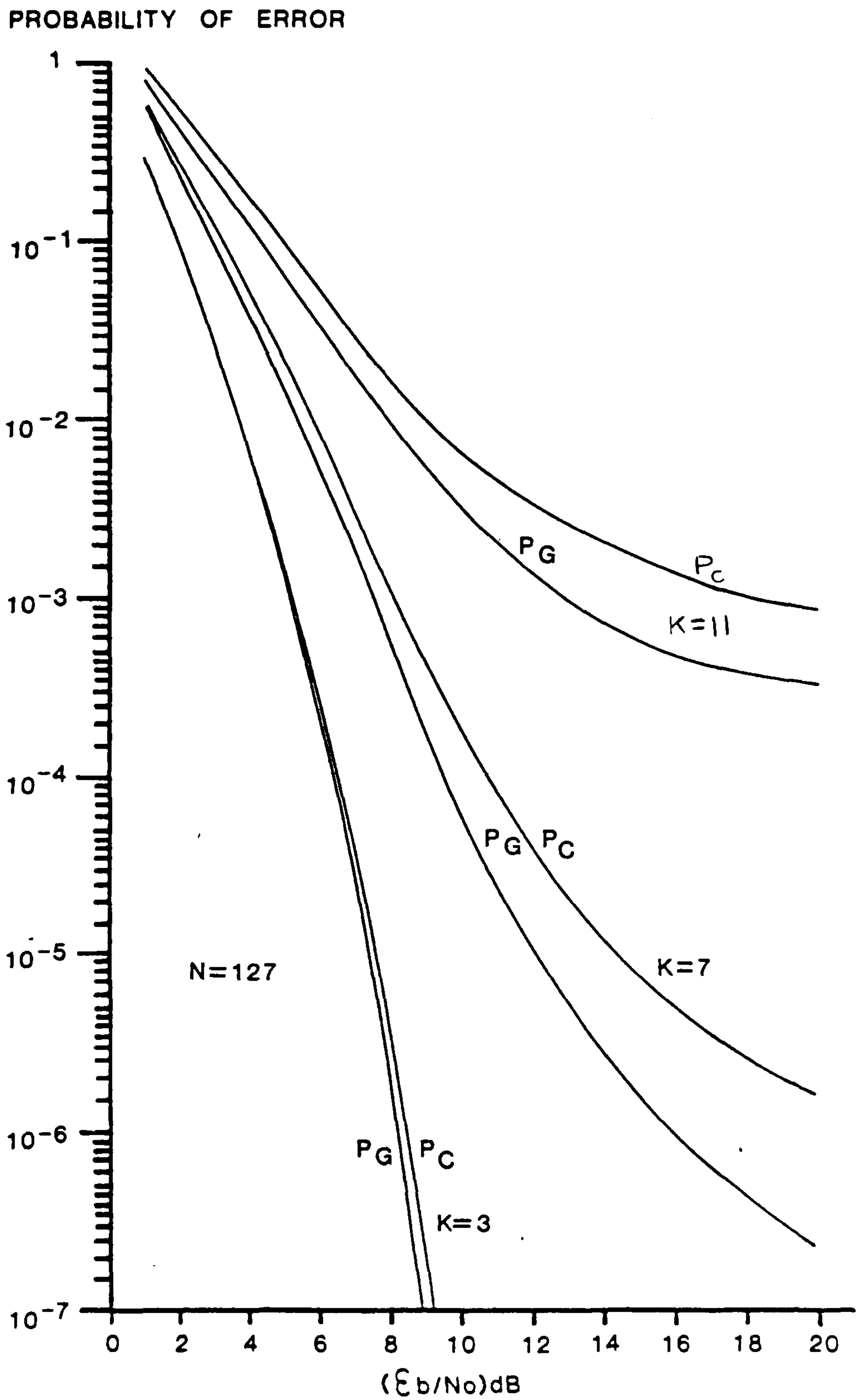


Figure 3.5 N=127 Multiuser Probability of Error

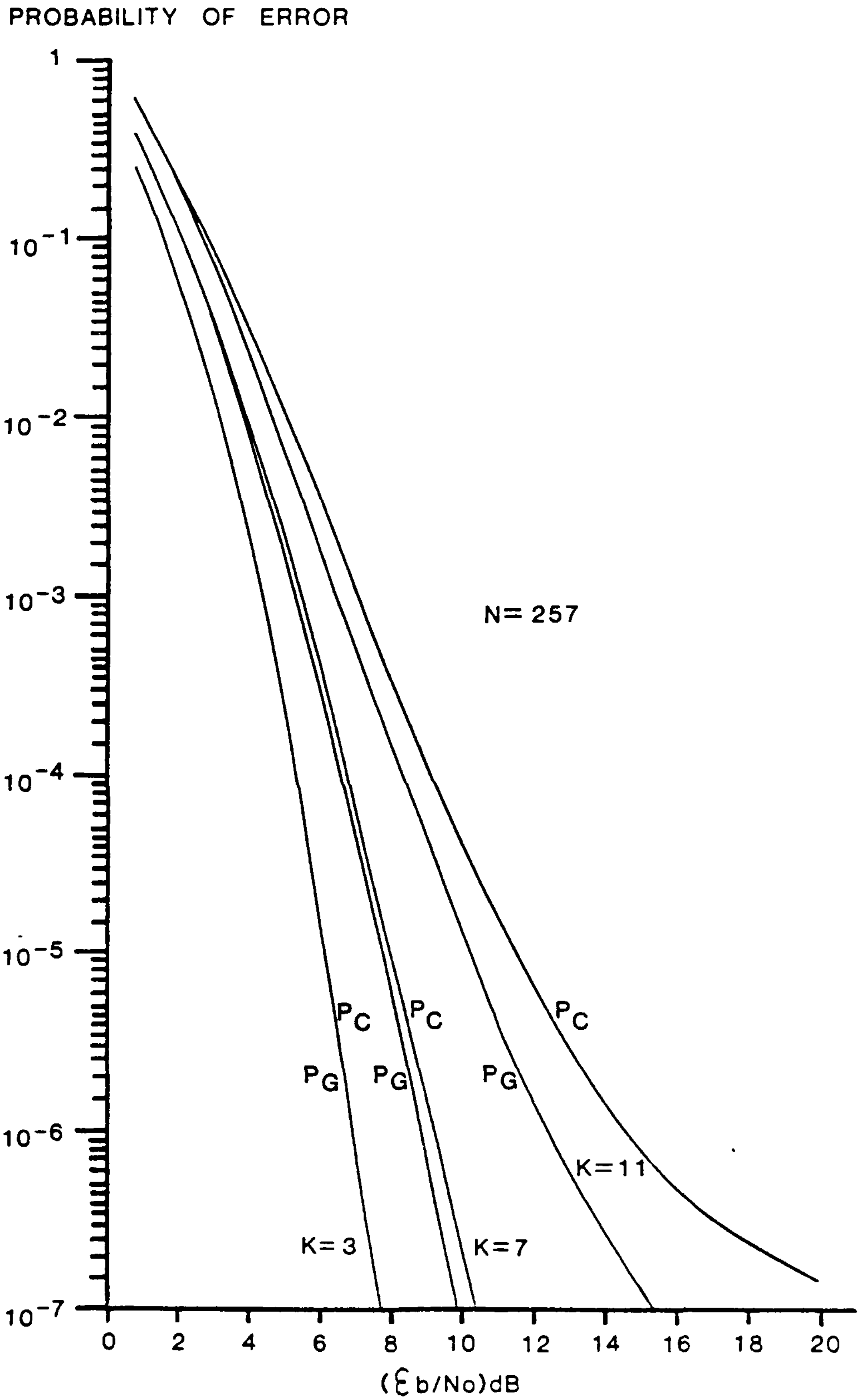


Figure 3.6 N=257 Multiuser Probability of Error

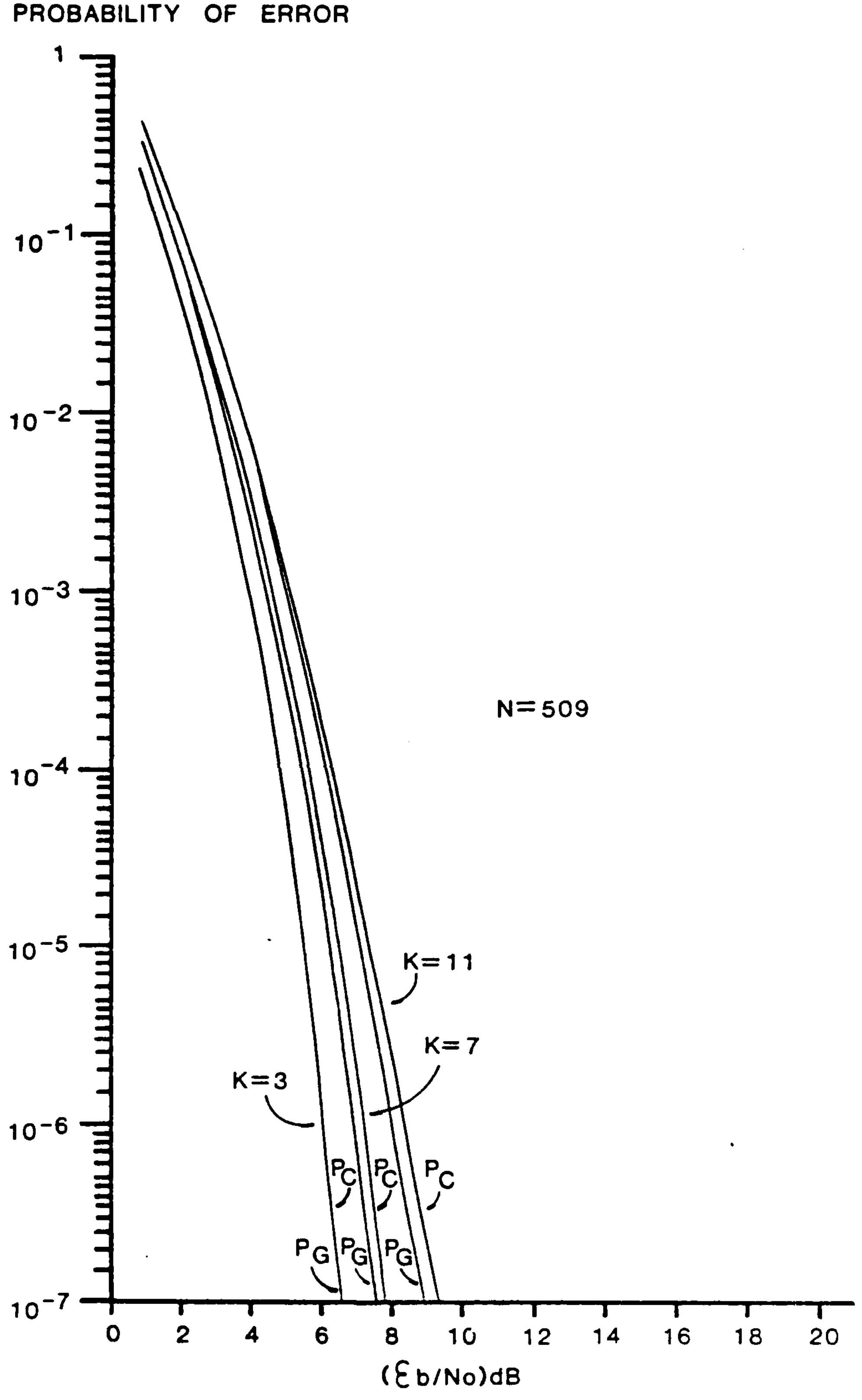


Figure 3.7 N=509 Multiuser Probability of Error

error of the $P_C$ curves is less than $1 \times 10^{-8}$, which is acceptable for the range of values plotted in Figures 3.3 through 3.7.

Moreover, we believe that the $P_C$ curves are very close to the actual MEP for probability of error less than .001. This accuracy results, because the union bound is very sharp in this region.

As shown in the figures, considerable discrepancy between the $P_C$ and $P_G$ curves is possible. Indeed, a difference of 4 dB exists at $Pr(\varepsilon)=4 \times 10^{-3}$ for K=3 and N=17. A slightly greater difference is shown at $Pr(\varepsilon)=1 \times 10^{-3}$ for K=11 and N=127. However, no difference is discernible for K=3, N=509 or K=3, N=257 curves. The discrepancy grows with increasing K or $\mathcal{E}_b/N_o$, because the relative magnitude of the multiple access interference is increasing. Correspondingly, the disagreement shrinks with increasing N, because of the decrease in the relative magnitude of the multiple access interference.

Before concluding this section, we turn our attention to a third technique for evaluating the multiuser probability of error. This technique is the same as our characteristic function/random sequence method except that we make the additional assumption that the $I_\mu^{(\alpha)}$ and $I_\lambda^{(\alpha)}$ are independent for all $\mu$ not equal to $\lambda$. With this additional assumption, the multiuser characteristic function becomes

$$\phi^{(\beta)}(\omega)=\left((1/T_c)\int_0^{T_c} (J_o(\omega\hat{R}(\tau)/2)J_o(\omega R(\tau)/2))^N d\tau\right)^2 \qquad (3.60)$$

This new expression was used with (3.59) to create multiuser probability of error curves and these curves were compared to our original $P_C$ curves, which were not based on an independence assumption. All differences between the two sets of curves were always less than the numerical error of our algorithm ($Pr(\varepsilon)<10^{-8}$). We observe that the $I_{\mu}^{(\alpha)}$ and $I_{\lambda}^{(\alpha)}$ are not independent, but an independence assumption does not produce meaningful probability of error discrepancies. This observation is consistent with the uncorrelatedness of $I_{\mu}^{(\alpha)}$ and $I_{\lambda}^{(\alpha)}$ for random coset codes and the nearly Gaussian nature of the multiuser interference pdf.

### 3.4.5 Alternative signalling schemes

In the last few subsections, we have used the Gaussian approximation and characteristic function approach to provide MEP expressions for our CDMA system, which uses r-phase modulation and orthogonal signal sets. These techniques can be used to obtain MEP results for a wide variety of signalling schemes, and in the next few paragraphs we will summarize results for the following systems:

- biphase modulation/orthogonal signal sets
- biphase modulation/antipodal signal sets
- r-phase modulation/antipodal signal sets.

Throughout the section, random coding analysis is used.

A CDMA system, which is identical to ours, except that it uses biphase modulation instead of r-phase modulation, can be analyzed. For biphase random coset codes, the second moment of the multiple access interference term is given by

$$E_c^{(\alpha)} E_c^{(\beta)} E_\tau^{(\beta)} E_b^{(\beta)} (I_\mu - I_\lambda)^2 = \frac{N \mathcal{M}_\Gamma}{2T_c} \tag{3.61}$$

Since this is identical to the second moment for our r-phase codes, both biphase and r-phase modulation result in the approximate MEP of Expression (3.46). Consequently, no large improvement in the <u>average</u> multiple access performance is realized through r-phase modulation.

Additionally, the MEP for the biphase/orthogonal system can be bounded with Expression (3.59). If we assume that the $I_\mu^{(\alpha)}$ and $I_\lambda^{(\alpha)}$ are independent and that the chip waveform is a slow function of time when compared to the carrier, then

$$\phi^{(\beta)}(\omega) = ((2/T_c\pi) \int_0^{T_c} \int_0^{\pi/2} ((\cos(\omega .5\hat{R}(\tau)\cos\theta)) \tag{3.62}$$

$$\cdot (\cos(\omega .5R(\tau)\cos\theta)))^N d\theta d\tau)^2$$

As discussed in the introductory chapter, a CDMA system which uses biphase modulation and antipodal signal sets has been analyzed by Pursley (1977), Pursley (1980), and Geraniotis (1982). Under the Gaussian approximation, the MEP for this system is

$$\Pr(\varepsilon) \approx Q\left[\left[\frac{N_o}{N\varepsilon_\Gamma} + \frac{\mathcal{M}_\Gamma}{NT_c\varepsilon_\Gamma^2}(K-1)\right]^{-\frac{1}{2}}\right] \tag{3.63}$$

From Geraniotis (1982), the MEP for this system that results from the characteristic function method is

$$Pr(\varepsilon)=Q((2\varepsilon_s/N_o)^{\frac{1}{2}})+ \tag{3.64}$$

$$(1/\pi)\int_0^{\infty} (1/\omega)\sin\omega \exp(-\omega^2 N_o/2N\varepsilon_\Gamma)(1-\prod_{\beta=2}^{K} \phi^{(\beta)}(2\omega/N\varepsilon_\Gamma))d\omega$$

where

$$\phi^{(\beta)}(\omega)=(2/T_c\pi)\int_0^{T_c}\int_0^{\pi/2} ((\cos(\omega .5\hat{R}(\tau)\cos\theta)) \times (\cos(\omega .5R(\tau)\cos\theta)))^N d\theta d\tau \tag{3.65}$$

For a system where the r-phase modulation is combined with antipodal signalling, the Expressions (3.63) and (3.64) still apply, but Expression (3.65) must be replaced with

$$\phi^{(\beta)}(\omega)=(1/T_c)\int_0^{T_c} (J_o(\omega\hat{R}(\tau)/2)J_o(\omega R(\tau)/2))^N d\tau \tag{3.66}$$

Equations (3.58) through (3.66) could be used to compare systems employing orthogonal signalling sets to antipodal signalling sets. Indeed, the probability of error would be less for the orthogonal system for a given K, $\varepsilon_b/N_o$ and rate. However, when rate is at a premium, the antipodal system should employ an "outer" error correcting code. This improvement muchly improves the performance of the antipodal system, but a comparison incorporating this embellishment is beyond our current scope. Additionally, the orthogonal system could also incorporate an "outer" code, where the outer code could be any variety of M symbol convolutional or block code. Once again, this improvement is beyond our current scope. In this section, we have used characteristic functions to derive the probability of error for CDMA systems using random sequence sets. In Appendix C, we consider the use of characteristic functions for estimating the probability of error for deterministic sequence sets.

# CHAPTER 4

# SEQUENCE DESIGN

## 4.1 Introduction

In this section, we present two sequence sets, which achieve the average performance of Chapter 3 (one is orthogonal and the other is nearly orthogonal) and have bounded worst case mutual interference. To do this, a worst case analysis of the multiple access interference is presented, and then a sequence design criterion is developed with the help of a result from number theory. Next, we will detour slightly to consider the LF multi-path interference problem and the signal acquisition problem because these phenomena may impose their own sequence design criteria. Finally, we present some designs based on additive and multiplicative characters.

## 4.2 Worst Case Analysis

The worst case strategy considers the interference to be a function of several deterministic variables. The resulting sequence design strategy minimizes the maximum value the magnitude of the interference can take. Accordingly, we seek an expression for

$$\max \left| I_{\lambda}^{(\alpha,\beta)} \right| \tag{4.1}$$

where the maximum is with respect ot user numbers, data sent, and interfering user delays. To make the analysis simpler we introduce some functions. First of all, the correlation between the signature sequence x and the overlap of the sequences u and v is (same as Equation 3.16)

$$H_{x,u,v}(\ell)=\sum_{n=\ell}^{N-1} x^{*}_{n-\ell}u_{n} + \sum_{n=0}^{\ell-1} x^{*}_{n+N-\ell}v_{n} \tag{4.2}$$

This function may be related to the aperiodic cross-correlation functions described in Sarwate and Pursley (1980) as follows

$$H_{x,u,v}(\ell)=C^{*}_{x,u}(\ell)+C_{v,x}(N-\ell) \tag{4.3}$$

Also note that

$$H_{x,u,u}(\ell)=\theta_{x,u}(\ell) \tag{4.4}$$

where $\theta_{x,u}(\ell)$ is the periodic cross-correlation function which is also extensively discussed in Sarwate and Pursley (1980). With the help of Definition (4.2), we may define the complex multiple access interference

$$\tilde{I}_{\lambda}=\hat{R}_{\Gamma}(\tau)\exp j\omega_{c}\tau\ H_{x_{\lambda},u,v}(\ell) + R_{\Gamma}(\tau)\exp j\omega_{c}\tau\ H_{x_{\lambda},u,v}(\ell+1) \tag{4.5}$$

Contrast this expression to the one for the real multiple access interference given by Equation (3.25) and note that

$$I_{\lambda}=\tfrac{1}{2}\mathrm{Re}\{\tilde{I}_{\lambda}\} \tag{4.6}$$

An upper bound on the maximum magnitude of the real multiple access interference may now be developed as follows (see Pursley (1982))

$$\max_{b,\tau} |I_\lambda| \le \max_{b,\tau} \tfrac{1}{2}|\tilde{I}_\lambda| \tag{4.7}$$

$$\le \max_{b,\tau} \tfrac{1}{2}(|\hat{R}_\Gamma(\tau)||\exp j\omega_c\tau| \; |H_{x_\lambda,u,v}(\ell)| +$$

$$|R_\Gamma(\tau)||\exp j\omega_c\tau| \; |H_{x_\lambda,u,v}(\ell+1)|)$$

$$=\max_{b,\tau} \tfrac{1}{2}(\hat{R}_\Gamma(\tau)|H_{x_\lambda,u,v}(\ell)| + R_\Gamma(\tau) \; |H_{x_\lambda,u,v}(\ell+1)|)$$

We observe that

$$0 \le \hat{R}_\Gamma(\tau) \le \varepsilon_\Gamma \tag{4.8}$$

$$0 \le R_\Gamma(\tau) \le \varepsilon_\Gamma \tag{4.9}$$

$$0 \le \hat{R}_\Gamma(\tau)+R_\Gamma(\tau) \le \varepsilon_\Gamma \tag{4.10}$$

These inequalities indicate that the possible values of $(R_\Gamma(\tau),\hat{R}_\Gamma(\tau))$ form a convex set. Furthermore, the right hand side of Inequality 4.7 is a linear function on that convex set. Consequently, the maximum magnitude of that linear function occurs at one of the corner points of the convex set. This means

$$\max_{b,\tau} |I_\lambda| \le \max_{u,v\in C^{(3)}} \; \max_{\ell\in\{0,\ \dots\ N-1\}} \tfrac{1}{2}\varepsilon_\Gamma |H_{x_\lambda,u,v}(\ell)| \tag{4.11}$$

For details, see Pursley (1982). The worst case interference given by Equation (4.11) could be minimized by minimizing the maximum member of the set

$$\{|H_{x,u,v}(\ell)|; x\varepsilon C^{(\alpha)}; u,v\varepsilon C^{(\beta)}; \alpha,\beta\varepsilon\{1,2\ \ldots\ K\}; \quad \alpha\neq\beta; \ell\varepsilon\{0,1\ \ldots\ N-1\}\} \tag{4.12}$$

A brute force search for the maximum member of the set given by (4.12) would require the evaluation of order $K^2M^3$ cross correlation functions, consequently, we consider Equation (4.3) and the Bound (4.11) to derive

$$\max_{b^{(\beta)},\tau^{(\beta)}} |I_\lambda| \leq \tag{4.13}$$

$$\tfrac{1}{2}\varepsilon_\Gamma \max_{u,v\ C^{(\beta)}} \max_{\ell\varepsilon\{0,\ldots N-1\}} (|C^*_{x,u}(\ell)|+|C_{v,x}(N-\ell)|)$$

This bound is, in turn, developed through the use of a partial sum theorem, which we discuss in the next section.

## 4.3 Partial Sum Theorem

In this section, we develop a partial sum theorem, so that we can bound the right hand side of the inequality (4.13). This bound will allow us to consider "full" sums instead of the awkward partial sums $C_{x,u}(\ell)$ or $C_{v,x}(N-\ell)$. We begin by defining a window sequence

$$w=\{w_m\}_{m=-\infty}^{\infty} \tag{4.14}$$

$$w_m=\begin{cases} 1 & m\varepsilon\{0,1\ldots N-1\} \\ 0 & \text{otherwise} \end{cases}$$

and by considering the sequences x and u to be infinite length sequences with periodicity N. Next, the associated ambiguity functions are defined as follows

$$\Delta_{w,w}(\ell,c) = \sum_{m=0}^{N-1} w_m w^{*}_{m+\ell} \exp(j2\pi cm/N) \tag{4.15}$$

$$= \sum_{m=0}^{N-\ell-1} \exp(j2\pi cm/N)$$

$$\Delta_{x,u}(\ell,c) = \sum_{n=0}^{N-1} x_n u^{*}_{n+\ell} \exp(j2\pi cn/N) \tag{4.16}$$

Observe that

$$\sum_{c=0}^{N-1} \Delta_{x,u}(\ell,c)\ \Delta^{*}_{w,w}(\ell,c) = \tag{4.17}$$

$$\sum_{n=0}^{N-1} x_n u^{*}_{n+\ell} \sum_{m=0}^{N-1} w_m w_{m+\ell} \sum_{c=0}^{N-1} \exp(j2\pi c(n-m)/N) =$$

$$N \sum_{n=0}^{N-1} x_n w_n u^{*}_{n+\ell} w_{n+\ell} = N\, C_{x,u}(\ell) =$$

$$\Delta_{x,u}(\ell,0)\ \Delta^{*}_{w,w}(\ell,0) + \Delta_{x,u}(\ell,1)\ \Delta^{*}_{w,w}(\ell,0) +$$

$$\ldots \quad + \Delta_{x,u}(\ell,N-1)\ \Delta^{*}_{w,w}(\ell,N-1)$$

$$= \theta_{x,u}(\ell)\ (N-\ell) + \sum_{c=1}^{N-1} \Delta_{x,u}(\ell,c)\ \Delta^{*}_{w,w}(\ell,c)$$

Consequently,

$$|NC_{x,u}(\ell)| \le (N-\ell)\ |\theta_{x,u}(\ell)| \tag{4.18}$$

$$+ \max_{c\neq 0}|\Delta_{x,u}(\ell,c)| \sum_{c=1}^{N-1} |\Delta^{*}_{w,w}(\ell,c)|$$

or

$$\max_{x,u,\ell} |C_{x,u}(\ell)| \le \max_{x,u,\ell} \{|\theta_{x,u}(\ell)| \quad (N-\ell)/N\} + \qquad (4.19)$$

$$\max_{x,u,\ell,c\neq 0} \{|\Delta_{x,u}(\ell,c)|\}(1/N) \max_{\ell} \sum_{c=1}^{N-1} \Delta^{*}_{w,w}(\ell,c)$$

To further develop the bound, we observe that

$$(1/N)\max_{\ell} \sum_{c=1}^{N-1} |\Delta^{*}_{w,w}(\ell,c)| = (1/N)\max_{\ell} \sum_{c=1}^{N-1} \left|\sum_{m=0}^{N-\ell-1} \exp(-j2\pi cm/N)\right| \quad (4.20)$$

$$= (1/N)\max_{\ell} \sum_{c=1}^{N-1} \left| \frac{\exp(-j2\pi c(N-\ell)/N)-1}{\exp(-j2\pi c/N)-1} \right|$$

$$= (1/N)\max_{\ell} \sum_{c=1}^{N-1} \frac{|\sin \pi c\ell/N|}{|\sin \pi c/N|}$$

Vinogradov (1949,p101) has shown that

$$(1/N)\max_{\ell} \sum_{c=1}^{N-1} \frac{|\sin \ell c\pi/N|}{|\sin \pi c/N|} \begin{cases} \le \ln N - (1/3)\ln(2[N/6]+1) & \forall N \\ \le (2/3)\ln N & N>40 \end{cases} \qquad (4.21)$$

where [N/6] denotes the greatest integer less than N/6. We have computed the left hand side of Equation (4.21) for N between 20 and 1000 and find that in this range, we have

$$(1/N)\max_{\ell} \sum_{c=1}^{N-1} \frac{|\sin \ell c\pi/N|}{|\sin \pi c/N|} \leq .4475 \ln N \qquad N \geq 20 \tag{4.22}$$

The coefficient of lnN decreases from .4475 as N increases from N=20 to N=1000. Consequently, we use the coefficient .4475 in what follows and we feel that this bound is valid for all N .cp 12 greater than 20. Combining these results with the Inequality (4.18) yields

$$\max_{x,u,\ell} |C_{x,u}(\ell)| \leq \max_{x,u,\ell} \{|\theta_{x,u}(\ell)|\ (N-\ell)/N\} + \tag{4.23}$$

$$.4475 \ln N \max_{x,u,\ell,c\neq 0} \{|\Delta_{x,u}(\ell,c)|\}$$

Additionally, we may now rewrite Inequality (4.13) as

$$\max_{x,u,v,\ell} |H_{x,u,v}(\ell)| \leq \max_{x,u,\ell} |\theta_{x,u}(\ell)| + \tag{4.24}$$

$$.895 \ln N \max_{x,u,\ell,c\neq 0} |\Delta_{x,u}(\ell,c)|$$

This approach, which is similar to an approach described by Lerner (1961), allows us to consider the function $\Delta_{x,u}(\ell,c)$ instead of the more difficult partial sum $C_{x,u}(\ell)$. Before turning our attention to the presentation of sequence set designs, we describe the effect that the LF multipath problem and the signal acquisition problem have on our sequence design strategy.

## 4.4 Low Frequency Multipath and Sequence Design

For most of the LF band, multipath is specular in nature and depends on the transmitter to receiver distance. It is a very important part of any LF channel model. For example, at 500 km, the multipath signal (skywave) can exceed the groundwave signal strength by tens of decibels at 100 kHz. On the other hand, at 100 km, the groundwave is generally at least 10 dB stronger than the skywave. For long ranges, skywave is problematic, because the delayed signal may resemble a signal other than the one currently being communicated. Using an analysis similar to the worst case multiple access analysis, it can be shown that the worst case skywave delay is equal to an integer number of chip times ($\ell_s T_c$). In such a situation, the interference introduced by skywave in the $\alpha^{th}$ receiver's $\lambda^{th}$ correlator is

$$\tfrac{1}{2}\mathcal{E}_T \mathrm{Re}\{H_{x,u,v}(\ell_s T_c)\} \qquad (4.25)$$

where $x=x_\lambda^{(\alpha)}$ and u and v are the previous and the current sequences, respectively, being sent by the $\alpha^{th}$ transmitter. Since the skywave delay will always be small, compared to the duration of any sequence transmission, the magnitude of the skywave interference may be conveniently bounded as follows

$$\tfrac{1}{2}\mathcal{E}_T |H_{x,u,v}(\ell_s)| \le \tfrac{1}{2}\mathcal{E}_T (|H_{x,v}(\ell_s)| + 2\ell_s) \qquad (4.26)$$

Skywaves of appreciable strength compared to groundwave are not delayed by more than 200 $\mu$sec, which for our model means that $\ell_s$ will not exceed 3. Hence, the maximum value of the following set is a reasonable skywave sequence design criterion

$$\{|\theta_{x,v}(\ell_s)|; x,v \epsilon C^{(\alpha)}; \ell_s \epsilon \{1,2,3\}\} \tag{4.27}$$

## 4.5 Signal Acquisition and Sequence Design

Signal acquisition is the process of making the correlators in the $\alpha^{th}$ receiver time synchronous with the reception of the $\alpha^{th}$ signal. For our phase coherent receivers, this process may be described as a 3 part procedure, where the parts are carrier, chip, and code word synchronization. Carrier synchronization is the process of bringing the receiver's carrier reference into phase with the received carrier. Correspondingly, chip synchronization is the procedure the receiver uses to align its chip reference time with the received chip waveforms. Many carrier and chip synchronization schemes are described in Stiffler (1971). Once carrier and chip synchronization have been achieved, the problem of code word or sequence synchronization remains. Code word synchronization is the problem of identifying the first element of the received sequences, or equivalently, determining whether the currently observed string of elements is a single complete sequence of an overlap of two sequences. If no external synchronization aid is available, this process may be enhanced by using comma codes or comma free codes. In this paper, comma codes will be considered.

In our case, comma coding will mean the transmission of a known member of the sequence set (the comma) at known intervals. Such a comma is called singular if the comma is guaranteed to be different from any code word/code word overlap. However, for noisy channels, comma coding schemes do not usually involve this constraint. Instead, the receiver sums the output of the correlator matched to the comma over several comma repetition intervals, and this sum becomes the decision statistic for the code word synchronization algorithm. If the comma is being observed, the mean of the sum increases linearly with each repetition interval. If a code word/code word overlap is being observed, the ratio of the sum's standard deviation to the synchronized mean can be made arbitrarily small with increasing observations. If a comma/code word overlap is being observed, the test also becomes increasingly reliable, but only if the comma is easily distinguished from shifted versions of itself. This requirement may be met through the minimization of the following sequence design criterion

$$\max_{\lambda} \min_{x} \{|C_{x,x}(\ell)|; x \varepsilon C^{(\alpha)}; \ell \varepsilon \{1,2...N-1\}\} \tag{4.28}$$

This criterion guarantees that, at least, one member of each sequence set is a suitable comma. In our analysis, bounds for the worst case $C_{x,x}(\lambda)$ in every code will be found by applying the partial sum theorem in a manner similar to the multiple access analysis. We may, finally, turn our attention to the description of specific sequence sets.

## 4.6 Sequence Designs Based on Additive Characters

In this section, sequence designs based on additive characters are considered. We will define additive characters, describe some of their fundamental properties, and state a useful additive character sum theorem. Next, we describe some additive character designs, which have been presented in the literature. Finally, we propose a sequence design for our LF/CDMA model and analyze its multiple access, signal acquisition and skywave performance.

An additive character $\psi$ is a map from the additive group of GF(p) to the complex numbers of unity magnitude, such that for all x and y in GF(p)

$$\psi(x+y)=\psi(x)\psi(y) \tag{4.29}$$

This definition implies that $\psi(0)=1$ and that every additive character on GF(p) is of the form

$$\psi_a(x)=\exp j2\pi ax/p \tag{4.30}$$

for some integer a. This expression is revealing, because it shows that sequence designs based on additive characters will employ p phases. Thus p should not be too large, because of implementation considerations. However, the following exponential sum theorem (Schmidt (1976)) allows us to prove some remarkable correlation results for these sequences. Let

$$g(x)=a_n x^n+a_{n-1}x^{n-1}+\ldots a_0 \tag{4.31}$$

be a polynomial with integer coefficients and $0<n<p$ and $p \nmid a_n$. Then

$$\left|\sum_{x \varepsilon GF(p)} \exp j2\pi g(x)/p\right| \leq (n-1)\sqrt{p} \tag{4.32}$$

Additive character sequences of period p (N=p) are considered in Frank and Zadoff (1973) and Chu (1972). If we limit consideration to their sequences of odd prime period, the $k^{th}$ element of one of their sequences is given by

$$v_k = \psi_1(ak^2) \qquad a\varepsilon\{1,2...p-1\} \tag{4.33}$$

These sequences have the ideal periodic auto-correlation function

$$\theta_{v,v}(\ell) = \begin{cases} N & \ell \equiv 0 \bmod N \\ 0 & \text{otherwise} \end{cases} \tag{4.34}$$

Also, the sequence definition may be generalized by the addition of a linear phase shift term without changing the auto-correlation behavior (Chu (1972)).

$$v_k = \psi_1(ak^2+bk) \qquad b\varepsilon\{0,1...p-1\} \tag{4.35}$$

Later on, sets of sequences were defined and analyzed by Sarwate (1979) and Alltop (1980). In their case, the $k^{th}$ element of the $a^{th}$ sequence is given by

$$v_{ak} = \psi_1(ak^2) \qquad a\varepsilon\{1,2...p-1\} \tag{4.36}$$

This set contains p-1 sequences of period p $(N=p)$ and the periodic correlation function obeys

$$|\theta_{v,v'}(\ell)| = \begin{cases} \sqrt{N} & v \neq v' \\ N & v=v', \ell \equiv 0 \\ 0 & v=v', \ell \not\equiv 0 \end{cases} \tag{4.37}$$

Additionally, sets of cubic phase sequences were considered by Alltop (1980), who defined

$$v_{ak} = \psi_1(k^3+ak) \qquad a \varepsilon \{1,2 \ldots p-1\} \tag{4.38}$$

These sequences are also periodic p, but their periodic correlation functions have slightly different characteristics

$$|\theta_{v,v'}(\ell)| = \begin{cases} N & v=v', \ell \equiv 0 \\ 0 & v \neq v', \ell \equiv 0 \\ \sqrt{N} & \text{otherwise} \end{cases} \tag{4.39}$$

A sequence set for use with our LF/CDMA model is now proposed. Let the $\lambda^{th}$ sequence (data $= \lambda$) in the code $C^{(\alpha)}$ be defined by

$$v_\lambda^{(\alpha)} = \{v_{\lambda,k}^{(\alpha)}\}_{k=0}^{p-1} \qquad \alpha \varepsilon \{1,2 \ldots p-1\} \;\; \lambda \varepsilon \{0,1 \ldots p-1\} \tag{4.40}$$

where

$$v_{\lambda,k}^{(\alpha)} = \psi_1(\alpha k^3+\lambda k) \tag{4.41}$$

This definition provides p-1 codes $(K \leq p-1)$ with p sequences $(M=p)$

in each code and every sequence has length or period p (N=p). Additionally, each user's code is a complete orthogonal set, because for all v and v'

$$\theta_{v,v'}(0) = \begin{cases} 0 & v \neq v' \\ N & v = v' \end{cases} \tag{4.42}$$

For the analysis of our design with respect to multiple access performance and comma coding perfomance, we consider the following function as recommended in Sections 4.3 and 4.4.

$$|\Delta_{v,v'}(\ell,c)| = |\sum_{k=0}^{N-1} v_k [v'_{k+\lambda}]^* \exp j2\pi ck/N| \tag{4.43}$$

$$= |\sum_{k\varepsilon GF(p)} \exp j2\pi (k^3(\alpha-\alpha')+k^2(-\alpha' 3\ell)+k(\lambda-3\alpha'\ell^2-\lambda'+c))/p|$$

For the multiple access case ($\alpha \neq \alpha'$), we apply the exponential sum theorem and find

$$|\Delta_{v,v'}(\ell,c)| \leq 2\sqrt{p} \tag{4.44}$$

We continue the analysis by applying the partial sum theorem as follows

$$|H^*_{x,u,v}(\ell)| \leq 2\sqrt{p} + 1.790 \ln N \sqrt{p} \tag{4.45}$$

This bound may, in turn, be used to approximately bound the maximum interference due to a single competing user as follows

$$\tfrac{1}{2} \text{[illegible]} \max_{u,v,\ell} |H_{x,u,v}(\ell)| \leq \text{[illegible]}\sqrt{N}(1+.895 \ln N) \tag{4.46}$$

This result does not guarantee that the worst case interference is less than the signal strength unless sequences of length exceeding 110 are used. Consequently, when considering multiple competing users, Equation (4.46) cannot be used to bound the $\alpha^{th}$ user's probability of error in any reasonable way. However, it does show that the worst case multiple access interference to signal strength ratio can be made arbitrarily small with increasing N and does provide a way for comparing different code set designs.

To consider the signal acquisition performance of our design, we apply the exponential sum theorem to Equation (4.43) with $\alpha=\alpha'$ and $\lambda=\lambda'$ and find

$$|\Delta_{v,v}(\ell,c)| \leq \sqrt{p} \tag{4.47}$$

Applying the partial sum theorem yields

$$|C_{v,v}(\ell)| \leq .4475 \sqrt{N} \ln N \tag{4.48}$$

This result is somewhat disappointing, because it shows that N must equal 110 before the worst case auto-correlation sidelobe to sequence length ratio equals 0.2. However, it does show that with increasing N, every sequence in every code can serve as a comma. The bound of (4.48) suggests that, at least, one very good comma can be found in each code, if some additional search scheme is designed.

As mentioned in Section 4.4, the skywave rejection performance of any single set may be bounded through evaluation

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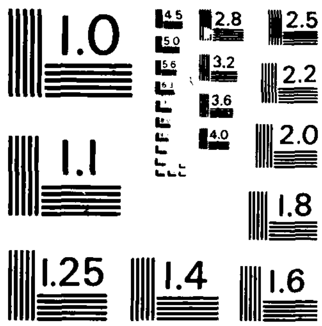


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of the worst case magnitude of the periodic cross-correlation function

$$|\theta_{v,v'}(\ell)| = \left|\sum_{k=0}^{N-1} v_k [v'_{k+\lambda}]^*\right| \tag{4.49}$$

$$= \left|\sum_{k\varepsilon GF(p)} \exp j2\pi (k^2(-\alpha' 3\ell) + k(\lambda - 3\alpha' \ell^2 - \lambda'))/p\right|$$

Applying the exponential sum theorem for $\ell \neq 0$, we find

$$\max_{\ell, v, v'} |\theta_{v,v'}(\ell)| \leq \sqrt{N} \tag{4.50}$$

This inequality along with (4.26) shows that the worst case skywave interference is bounded by

$$\tfrac{1}{2}\mathcal{E}_T \max_{x,u,v,\ell} |H_{x,u,v}(\ell)| \leq \tfrac{1}{2}\mathcal{E}_T(\sqrt{N}+6) \tag{4.51}$$

If the received skywave power equals the received groundwave power, then the demodulated skywave to groundwave voltage ratio will not exceed 0.2, if the sequence length exceeds 25.

Our additive character sequence design ($\psi$-design) has now been evaluated with respect to worst case multiple access, comma coding and skywave performance. It can be compared to other designs through these bounds, but the greatest liability of the design may be that it requires N phases. Since this requirement will make it difficult to realize designs with long sequences, we turn our attention to designs based on multiplicative characters.

This result does not guarantee that the worst case interference is less than the signal strength unless sequences of length exceeding 110 are used. Consequently, when considering multiple competing users, Equation (4.46) cannot be used to bound the $\alpha^{th}$ user's probability of error in any reasonable way. However, it does show that the worst case multiple access interference to signal strength ratio can be made arbitrarily small with increasing N and does provide a way for comparing different code set designs.

To consider the signal acquisition performance of our design, we apply the exponential sum theorem to Equation (4.43) with $\alpha=\alpha'$ and $\lambda=\lambda'$ and find

$$|\Delta_{v,v}(\ell,c)| \leq \sqrt{p} \tag{4.47}$$

Applying the partial sum theorem yields

$$|C_{v,v}(\ell)| \leq .4475 \sqrt{N} \ln N \tag{4.48}$$

This result is somewhat disappointing, because it shows that N must equal 110 before the worst case auto-correlation sidelobe to sequence length ratio equals 0.2. However, it does show that with increasing N, every sequence in every code can serve as a comma. The bound of (4.48) suggests that, at least, one very good comma can be found in each code, if some additional search scheme is designed.

As mentioned in Section 4.4, the skywave rejection performance of any single set may be bounded through evaluation

## 4.7 Sequence Design Based on Multiplicative Characters

As mentioned at the end of the last section, multiplicative character sequences are now considered, because they will require fewer than N phases. In this section, we begin by defining multiplicative characters, discussing their properties, and stating two character sum theories that will allow the calculation of correlation bounds. Then, we discuss sequence sets, which have appeared in the literature, and finally, we propose a multiplicative character sequence design for our LF CDMA model.

Let GF(p) denote the finite field of order p and $GF^*(p)$ denote the multiplicative group of GF(p), consisting of the non-zero elements of GF(p). Additionally, let g denote a generator for $GF^*(p)$, such that

$$x=g^{I(x)} \tag{4.52}$$

for all x in $GF^*(p)$ and where I(x) is the index of x. The multiplicative character on $GF^*(p)$ is a map $\chi$ from $GF^*(p)$ to the complex numbers with unity magnitude such that

$$\chi(xy)=\chi(x)\chi(y) \tag{4.53}$$

for all x and y in $GF^*(p)$. This definition implies that $\chi(1)=1$ and that every multiplicative character on $GF^*(p)$ is of the form

$$\chi_a(x)=\exp j2\pi aI(x)/(p-1) \tag{4.54}$$

Every character will have

$$\chi_a^{p-1}=\chi_0 \tag{4.55}$$

where $\chi_o$ is the principal character, which means $\chi_o(x)=1$ for all x. The order of $\chi_a$ is the smallest positive integer d such that $\chi_a{}^d= \chi_o$. The order of $\chi_a$ equals (p-1)/gcd(a,p-1). The order is a particularly important parameter in sequence design, because it is equal to the number of phases used by the sequence. The definition of multiplicative characters may be extended as follows to map all of GF(p) to the complex plane

$$\chi(0)= \begin{cases} 1 & \chi=\chi_0 \\ 0 & \chi\neq\chi_0 \end{cases} \tag{4.56}$$

Additionally, the following identities hold

$$\chi_a^*(x)=[\chi_a(x)]^{p-2} \tag{4.57}$$

$$\chi_{a'}(x)=[\chi_a(x)]^{\tilde{a}}$$

where $a'=a\tilde{a}$ mod p-1. Two results on character sums, which will help us find correlation results, are now presented. The first of these is due to Sidel'nikov (1969).

$$\sum_{x\varepsilon GF(p)} \chi_a[(x+c)^i(x+c')^{d-i}]=-1 \tag{4.58}$$

where $0<i<d$, $c \neq c'$ and d is the order of $\chi_a$. The second result

is presented in Schmidt (1976) and bounds the magnitude of a multiplicative character sum. Let $\chi$ be a multiplicative character of order $d>1$, and let $f(x)$ be an element of $GF(p)[x]$ ($GF(p)[x]$ is the ring of polynomials with coefficients from $GF(p)$.) with m distinct zeroes. Additionally, $f(x)$ is not a $d^{th}$ power, which means it cannot be expressed as $f(x)=c(\lambda(x))^d$ where c is in $GF(p)$ and $\lambda(x)$ is in $GF(p)[x]$, then,

$$\left| \sum_{x \in GF(p)} \chi(f(x)) \right| \leq (m-1)\sqrt{p} \qquad (4.59)$$

We now describe some multiplicative character sequence designs, which have been presented in the literature.

Length p-1 sequences are described in Lerner (1961) and similar sequences of any length are defined by Scholtz and Welch (1978). We will describe the length p-1 sequences only, because the value of the extension for our LF/CDMA model is not clear. A sequence of length p from a set of p-2 sequences may be defined as follows

$$v_a=\{v_{ak}\}_{k=0}^{p-1} \qquad a \in \{1,2...p-1\} \qquad (4.60)$$

where $v_{ak}=\chi_a(k)$ and $v_{a0}=0$. The periodic auto-correlation function for any of these sequences is

$$\theta_{v,v}(\ell) = \begin{cases} p-1 & \ell \equiv 0 \bmod p \\ -1 & \ell \not\equiv 0 \bmod p \end{cases} \qquad (4.61)$$

The magnitude of the periodic cross-correlation function is given by

$$|\theta_{v,v'}(\ell)| = \begin{cases} \sqrt{p} & \ell \not\equiv 0 \bmod p \\ 0 & \ell \equiv 0 \bmod p \end{cases} \tag{4.62}$$

This sequence set is nearly ideal with respect to simultaneous minimization of the maximum value of the periodic auto-correlation and cross-correlation functions (Sarwate (1979)).

A single sequence design, which has a nearly ideal periodic auto-correlation function is given by Sidelnikov (1969). Limiting our interest to GF(p), Sidelnikov's sequences are defined as follows

$$v=\{v_k\}_{k=0}^{p-2} \tag{4.63}$$

$$v_k = \begin{cases} \chi(g^k+1) & g^k+1 \not\equiv 0 \bmod p-1 \\ 1 & g^k+1 \equiv 0 \bmod p-1 \end{cases}$$

These sequences have period p-1 (N=p-1) and $v_k$ is defined to be 1 when $g^k+1$ is congruent to zero to provide uniform envelope signals. Sidelnikov proves that

$$|\theta_{v,v}(\ell)| \leq 4 \qquad \ell \not\equiv 0 \bmod N \tag{4.64}$$

Additionally, if the order of the character is 2 (providing biphase sequences), then

$$|\theta_{v,v}(\ell)| = \pm 2 \qquad \ell \not\equiv 0 \bmod N \tag{4.65}$$

A sequence set design, based on multiplicative characters, (Krone and Sarwate (1982)) uses the definition

$$v_c=\{v_{c,k}\}_{k=0}^{p-2} \qquad c\varepsilon\{0,1...p-1\} \tag{4.66}$$

$$v_{c,k} = \begin{cases} \chi(g^{2k}+2g^{k}+c) & \text{if } g^{2k}+2g^{k}+c\neq 0 \\ 1 & \text{if } g^{2k}+2g^{k}+c=0 \end{cases}$$

This definition provides p sequences of period p-1 and it may be shown that

$$|\theta_{v,v'}(\ell)| \begin{cases} \leq 3\sqrt{p} + 5 & \ell\neq 0 \text{ or } c\neq c' \\ = p-1 & \ell=0 \text{ and } c=c' \end{cases} \tag{4.67}$$

This design is powerful, because the number of phases employed by all p sequences is equal to the order of the character. Hence, a sequence set for quadriphase signalling could be designed by choosing a and p such that gcd(a,p-1)=(p-1)/4.

We now propose a sequence design for our LF CDMA model based on multiplicative characters. We show that each user's set is nearly orthogonal and then analyze multiple access, signal acquisition and skywave performance. Let the $\lambda^{th}$ code word of $C^{(\alpha)}$ be defined by

$$v_{\lambda}^{(\alpha)}=\{v_{\lambda,k}^{(\alpha)}\}_{k=0}^{p-2} \qquad \alpha\varepsilon\{1,2...p-2\} \qquad \lambda\varepsilon\{1,2...p-2\} \tag{4.68}$$

where

$$v_{\lambda,k}^{(\alpha)}=\chi_{\alpha}(g^{k}+\lambda) \tag{4.69}$$

This design provides p-2 sets of sequences (K=p-2), where each set contains p-2 sequences (M=p-2) of length or period p-1 (N=p-1). We do not make an effort to insure that every element has unity magnitude, because uniform envelope signals are not necessarily required for the LF channel.

As shown by the definition, the user number ($\alpha$) determines the order of the character and, consequently, the number of phases required by that user. Consequently, if p-2 users (K=p-2) were operating, then $\phi$(p-1) of these would use p-1 phases where $\phi$ is Euler's totient function. These $\phi$(p-1) users would be using N phases and it would appear that the design had failed to overcome the main drawback of additive character designs. However, all CDMA systems must have much fewer than N active users, based on average performance analysis. For those systems without large numbers of inactive potential users, the sequence length can be chosen such that all users can be accommodated without requiring any to use a large number of phases. For example, if we choose p=257 (and consequently N=256) the user/phase breakdown for 31 users could be

| number of users | number of phases |
|---|---|
| 2 | 4 |
| 4 | 8 |
| 8 | 16 |
| 16 | 32 |

In general, the choice of the user numbers ($\alpha$) must be constrained in a special way to guarantee good multiple access performance. Our design requires that $\alpha$ be even and that if $\alpha = \tilde{\alpha}\alpha'$ mod p-1 for any two codes $\alpha$ and $\alpha'$, then $\tilde{\alpha}$ is even.

We may show that each individual signal set $C^{(\alpha)}$ is nearly orthogonal as follows

$$\theta_{v,v'}(0) = \sum_{k=0}^{p-2} v_k [v_k']^* \qquad (4.70)$$

$$= \sum_{k\varepsilon GF(p)} \chi_\alpha [(g^k+\lambda)(g^k+\lambda')^{d-1}] - v_{p-1}[v_{p-1}']^*$$

Using Sidelnikov's results

$$\theta_{v,v'}(0) = -1 - v_{p-1}[v_{p-1}']^* \qquad (4.71)$$

Hence,

$$|\theta_{v,v'}(0)| \leq 2 \qquad v,v' \varepsilon C^{(\alpha)};v \neq v' \qquad (4.72)$$

We now evaluate the designs multiple access performance by using the function

$$|\Delta_{v,v'}(\lambda,c)| = |\sum_{k=0}^{N-1} v_k v^*_{k+\lambda} \exp j2\pi ck/N| \tag{4.73}$$

$$\leq |\sum_{k\varepsilon GF(p)} \chi_{\alpha'}[(g^k+\lambda)^{\tilde{\alpha}}(g^k+\lambda' g^{-\ell})(g^k)^{\tilde{c}}]| +1$$

where $\alpha=\tilde{\alpha}\,\alpha'$ mod p-1 and $\tilde{c}=c\alpha'$ mod p-1. Applying the sum theorem for $\tilde{\alpha}\neq 1$, it can be shown that

$$\max_{c\varepsilon\{1,2...N-1\}} |\Delta_{v,v'}(\ell,c)| \leq 2\sqrt{p} +1 \tag{4.74}$$

$$|\Delta_{v,v'}(\ell,0)| \leq \sqrt{p} +1$$

Consequently,

$$\max_{u,v,\ell} \tfrac{1}{2}\varepsilon_\Gamma \; |H_{x,u,v}(\ell)| \leq \tfrac{1}{2}\varepsilon_\Gamma((\sqrt{N+1} +1)+.895 \ln N \; (2\sqrt{N+1} +1)) \tag{4.75}$$

This final bound on the maximum multiple access interference for our multiplicative character sequence set ($\chi$ set) is approximately the same as the bound for our $\psi$ set.

We analyze the signal acquisition of the $\chi$ set by setting $\tilde{\alpha}=1$ and $\lambda=\lambda'$ in Equation (4.73) and again applying the sum theorem, which gives us

$$|\Delta_{v,v'}(\lambda,c\neq 0)| \leq 2\sqrt{p} +1 \qquad \ell\neq 0 \tag{4.76}$$
$$= p \qquad \ell\equiv 0$$

$$|\Delta_{v,v'}(\lambda,c=0)| \leq \sqrt{p} +1 \qquad \ell\neq 0$$
$$= p \qquad \ell\equiv 0$$

These bounds yield

$$|C^{*}_{v,v}(\ell)| \leq (\sqrt{N+1} + 1) + .4475 \ln N \, (2\sqrt{N+1} + 1) \tag{4.77}$$

This performance bound is approximately twice that obtained for our additive character design, but the same discussion applies.

Finally, to investigate skywave performance

$$|\theta_{v,v'}(\ell)| \leq \left| \sum_{k \in GF(p)} \chi_{\alpha'} [(g^k+\lambda)(g^k+\lambda' g^{-\ell})^{p-2}] \right| + 1 \tag{4.78}$$

Applying the sum theorem yields

$$|\theta_{v,v'}(\ell)| \begin{array}{ll} \leq \sqrt{N+1} & \lambda \not\equiv \lambda' g^{-\ell} \\ \leq N & \lambda = \lambda' g^{-\ell} \end{array} \tag{4.79}$$

As shown, the average skywave interference for our $\chi$ design cannot be reasonably bounded, but each code may be altered as follows to provide good skywave performance. For each $x_\lambda^{(\alpha)}$ in $C^{(\alpha)}$ expurge the set

$$\{x_\mu^{(\alpha)}; \mu = \lambda g^{-\ell}; \ell \epsilon \{1,2 \ldots \ell_s\}\} \tag{4.80}$$

The new code will have worst case skywave interference bounded by $(1/2)\, \varepsilon_\Gamma (\sqrt{N} + 6)$, but it also has reduced rate.

In this section, a design that can employ many fewer than N phases has been discussed. In the final chapter of this thesis, we will summarize these results.

# CHAPTER 5

# SUMMARY AND CONCLUSIONS

In this thesis, we have proposed and analyzed a code division multiple access communication system, which is especially suited for the low frequency channel.

Our CDMA scheme is similar to classical CDMA systems in the following respects. It has K users sharing a channel by phase modulating their transmissions with signature sequences. These users make no attempt at frequency separation and the scheme is asynchronous, which means that the users are not time coordinated in any way.

However, our scheme is more complicated than the classical systems in the following respects. Each user has a sequence set, which consists of M orthogonal sequences. This change increases the complexity of the system, but now $\log_2 M$ bits of information are transmitted by choosing among the signature sequences. Moreover, for a given bandwidth and communication reliability, the information rate is greater than that achieved with traditional antipodal sequence sets.

Additionally, our scheme employs r-phase modulation (the signature sequences are r-valued). This change also represents an increase in complexity, but probably provides an improvement in the worst case multiple access performance of the system. Indeed, both of the sequence designs we found for our LF/CDMA communication system involved r-valued sequences.

Our analysis did not model each user's initial carrier phase as random, because LF transmitter techniques are such that the initial carrier phase is fixed and stable with respect to the signal envelope. However, if the initial carrier phase was modelled as a uniform random variable on $[0,2\pi)$, our system analysis results would not change. This invariance exists, because we assumed the modulation phase of each chip was uniform on $[0,2\pi)$. The invariance would also result for biphase modulation if the chip envelope varied slowly compared to the carrier.

Finally, our model includes an additive, impulsive noise source to represent LF atmospheric noise. We used a truncated Cauchy probability distribution function to represent the first order pdf of this atmospheric noise after filtering and sampling. The strongly non-Gaussian nature of these noise samples leads us to derive and analyze a locally optimum Bayes detector (LOBD), which is based on our noise model. The derived receiver consists of a filter matched to the carrier modulated by the chip waveform, followed by a sampler. The samples are then processed by a zero memory nonlinearity and then fed to a bank of M matched filters, where each filter corresponds to one of the M members of the sequence set. The performance of this structure is a strong function of Fisher's information number, which in turn depends entirely on the statistics of the sampled noise. By evaluating Fisher's information number for our combined Gaussian, Cauchy and multiple access noise, we found that every LF/CDMA receiver should employ a ZNL to limit the effects of the impulsive events.

We also found that such a nonlinear receiver performs approximately as well as a linear receiver operating in an environment without the Cauchy noise.

Consequently, we analyzed the single and multiuser performance of a linear receiver with additive white Gaussian noise. The multiuser analysis is approached through a Gaussian assumption and a characteristic function method. The former technique is based on the assumption that the multiuser interference has a Gaussian pdf and leads to an approximate multiuser error probability (MEP) expression. The approximation is conveniently simple, but our characteristic function analysis indicates that it does not always give accurate results. The difference between the results of the two approaches increases with the magnitude of the multiple access noise relative to the atmospheric noise. The results of both analyses are summarized in Figures 3.3 through 3.7.

The multiple access performance of our LF/CDMA model can be appreciated by considering the following example. Suppose, we wish to achieve a MEP of approximately $10^{-5}$ and we intend to use chip waveforms of 100 sec duration, which corresponds to a bandwidth of approximately 20 kHz. Then a scheme with sequences of length 17 will accommodate 2 users and an energy per bit to noise density ratio of no less than 16 dB. Each user will be able to communicate at an information rate of 2400 bits per second. On the other hand, a scheme with N=257 will accommodate 3,7, or 11 users with $\mathcal{E}_b/N_o$ greater than or equal to 6 dB, 8 dB, or 12 dB respectively. The information rate would be slightly greater than 300 bits per second.

In Chapter 4 of the thesis, we developed a sequence set design technique for our LF/CDMA system and used it to synthesize 2 sequence sets.

The sequence set design strategy is to minimize the maximum magnitude of the interference that any one user's transmitter can introduce into the receiver of another user. Accordingly, we bound the maximum value this interference can take as a function of the interferer's delay and choice of sequences. However, evaluation of this initial bound requires consideration of all the aperiodic cross-correlation functions between the sequences of the two users. Such an evaluation by computer would be prohibitively time consuming and direct analysis is difficult, because aperiodic cross-correlation functions are partial sums, for which few results exist in the literature. Consequently, we considered a partial sum theorem (Vinogradov, 1949), which allowed us to bound the magnitude of partial sums in terms of the magnitude of the more convenient and widely studied full sums (Vinogradov (1949), Lerner (1961)). As such , we developed a multiple access sequence design criterion, which depends on analytically tractrable full sums (ambiguity functions).

Even though multiple access interference is the main concern of the thesis, skywave (specular multipath) and signal acquisition are major issues in the design of LF communications systems. Consequently, we also considered these topics briefly in Chapter 4 and developed appropriate sequence design criteria.

Once our sequence design strategy was clear, we proceeded to consider actual designs based on additive and multiplicative characters in Sections 4.6 and 4.7 respectively. In both sections, we began by reviewing appropriate definitions and theorems. Most importantly, we reviewed theorems which allowed us to bound full sums of additive and multiplicative characters. These full sum theorems along with the partial sum theorem discussed above allowed us to bound the multiple access, skywave and signal acquisition performance of our sequence sets. In both sections, we also presented a brief history of the use of additive and multiplicative characters for code or sequence set design. Finally, we presented an additive and multiplicative character sequence set design for our LF/CDMA communication system. The main difference between the two sequence designs may be the number of phases used to modulate the user transmissions. The additive character design accommodates N users, but requires N phases, where N is the sequence length. The multiplicative character designs can use many fewer than N phases, but consequently accommodates many fewer than N users.

# APPENDIX A

# DERIVATION OF THE VARIANCE OF MULTIPLE ACCESS INTERFERENCE

## A.1 Introduction

In this appendix, we will derive the variance (or equivalently the second moment) of the multiple access interference found in our r-phase/orthogonal signal set CDMA system. We have already stated the result in Equation (3.37) and used it to derive many important insights in Chapter 3. Mathematically, we seek an expression for

$$E_b^{(\beta)} E_\tau^{(\beta)} \ (I_\mu - I_\lambda)^2 \tag{A.1}$$

and our search is broken into the following sections.

- Define convenient symbology.
- Show relationship between variance of real interference and complex interference.
- Find expression for variance of complex interference.
- Find expression for variance of real interference.

## A.2 Definitions and Symbols

An expression for $I_{\lambda}^{(\alpha)}$ is given by Equation (3.25) and that expression is repeated below except that the notation is streamlined for simplicity

$$I_{\lambda} = \tfrac{1}{2}\hat{R}(c\ \mathrm{Re}\{H_y(\ell)\} - s\ \mathrm{Im}\{H_y(\ell)\}) \tag{A.2}$$

$$+ \tfrac{1}{2}R(c\ \mathrm{Re}\{H_y(\ell+1)\} - s\ \mathrm{Im}\{H_y(\ell+1)\})$$

where $x_{\mu}=y$ is the $\mu^{th}$ sequence and $x_{\lambda}=y$ is the $\lambda^{th}$ sequence and where

$$\hat{R}=\hat{R}_{\Gamma}(\tau^{(\beta)}) \tag{A.3}$$

$$R=R_{\Gamma}(\tau^{(\beta)})$$

$$c=\cos\omega_c\tau^{(\beta)}$$

$$s=\sin\omega_c\tau^{(\beta)}$$

$$H_y(\ell)=H_{x_{\lambda},u,v}(\ell)$$

An expression for complex multiple access interference is now defined which uses similarly streamlined notation.

$$\tilde{I}_{\lambda}=\hat{R}\ e\ H_y(\ell) + R\ e\ H_y(\ell+1) \tag{A.4}$$

where

$$e = \exp j\omega_c \tau^{(\beta)} \tag{A.5}$$

## A.3 Relationships Between Real and Complex Interference

By manipulating Expressions (A.2) and (A.4), the following relationships can be derived

$$I_\lambda = \tfrac{1}{2}\mathrm{Re}\{\tilde{I}_\lambda\} \tag{A.6}$$

$$|I_\lambda| \leq \tfrac{1}{2}|\tilde{I}_\lambda| \tag{A.7}$$

$$E_b\{|I_\lambda|^2\} \leq \tfrac{1}{4} E_b\{|\tilde{I}_\lambda|^2\} \quad \text{for all } \tau \tag{A.8}$$

Additionally, for systems where the carrier frequency varies quickly when compared to the chip waveform

$$E_\tau\{|\tilde{I}_\mu - \tilde{I}_\lambda|^2\} = 8E_\tau\{(I_\mu - I_\lambda)^2\} \tag{A.9}$$

The last equality relates the second moment of complex multiple access interference to the second moment of real interference. It is the most important of the above expressions and will be further developed in the next section.

## A.4 Second Moment of Complex Interference

In this section of the appendix, we seek an expression for the second moment of the complex multiple access interference

$$E_b E_\tau \{ | \tilde{I}_\mu - \tilde{I}_\lambda |^2 \}$$

This search is made convenient by proving the following two lemmas.

First of all, we wish to show that

$$(1/M) \sum_{u \varepsilon C^{(\beta)}} u_k u_i^* = \delta_{k-i} \tag{A.10}$$

where $u_k$ and $u_i$ are elements of the vector u, which belongs to an orthogonal code $C^{(\beta)}$. Consider that an orthogonal code is a set of vectors where

$$(1/N) \sum_{n=0}^{N-1} u_n v_n^* = \begin{cases} 0 & u \neq v \\ 1 & u = v \end{cases} \tag{A.11}$$

where $u_n$ and $v_n$ are vector elements. Equation (A.11) may be written in matrix form as follows

$$BB^{*T} = NI \tag{A.12}$$

where B is a matrix whose rows are the codewords and I is the identity matrix. The left hand side of Equation (A.10) may be rewritten

$$(1/M) \sum_{u \varepsilon C^{(\beta)}} u_k u_i^* = B^T B^* \tag{A.13}$$

but clearly

$$(1/M) \sum_{u\varepsilon C(\beta)} u_k u_i^* = B^T B^* = (BB^{*T})^T = NI = MI \tag{A.14}$$

which proves our assertion.

The second lemma we wish to prove is that

$$E_b\{H_x(\ell+a')H_y^*(\ell+a)\} = \begin{cases} C_{y,x}(a-a') & a \le a' \\ C^*_{x,y}(a'-a) & a' > a \end{cases} \tag{A.15}$$

We proceed as follows

$$E_b\{H_x(\ell+a')H_y^*(\ell+a)\} = \tag{A.16}$$

$$E_b\{(C^*_{x,u}(\ell+a')+C_{v,x}(N-\ell-a'))(C_{y,u}(\ell+a)+C^*_{v,y}(N-\ell-a))\}$$

$$=(1/M)\sum_u C^*_{x,u}(\ell+a')C_{y,u}(\ell+a) + (1/M^2)\sum_u\sum_v C^*_{x,u}(\ell+a')C^*_{v,y}(N-\ell-a)$$

$$+(1/M^2)\sum_u\sum_v C_{v,x}(N-\ell-a')C_{y,u}(\ell+a)$$

$$+(1/M)\sum_v C_{v,x}(N-\ell-a')C^*_{v,y}(N-\ell-a)$$

$$=(1/M)\sum_{k=\ell+b}^{N-1} x^*_{k-\ell-b}\sum_{i=\ell+a}^{N-1} y_{i-\ell-a}\sum_u u_k u_i^* + 0 + 0$$

$$+(1/M)\sum_{k=0}^{\ell+b-1} x^*_{k+N-\ell-b}\sum_{i=0}^{\ell+a-1} y_{i+N-\ell-a}\sum_v v_k v_i^*$$

If we substitute in our result from the previous lemma, which is stated in Equation (A.10), we may derive

$$E_b\{H_{x,u,v}(\ell+a')H_{y,u,v}(\ell+a)\} \tag{A.17}$$

$$= \begin{cases} \sum_{i=\ell+a}^{N-1} y_{i-\ell-a}x^*_{i-\ell-b} + \sum_{i=0}^{\ell+b-1} x^*_{i+N-\ell-b}y_{i+N-\ell-a} & a \geq b \\ \sum_{i=\ell+b}^{N-1} x^*_{i-\ell-b}y_{i-\ell-a} + \sum_{i=0}^{\ell+a-1} y_{i+N-\ell-a}x^*_{i+N-\ell-b} & b > a \end{cases}$$

$$= \begin{cases} C_{y,x}(a-a') & a \geq a' \\ C^*_{x,y}(a'-a) & a' > a \end{cases}$$

This final equation is the sought after result for the second lemma.

With the aid of Equation (A.15), we may now conveniently derive an expression for the second moment of the complex interference. We begin as follows

$$E_bE_\tau\{|I_\mu-I_\lambda|^2\} \tag{A.18}$$

$$=E_bE_\tau\{|I_\mu|^2+|I_\lambda|^2-2\mathrm{Re}\{I_\mu I^*_\lambda\}\}$$

$$=E_b(1/T)\sum_{\ell=0}^{N-1} (\mathcal{M}_\Gamma(|H_x(\ell)|^2+|H_x(\ell+1)|^2+|H_y(\ell)|^2+|H_y(\ell+1)|^2-$$

$$2\mathrm{Re}\{H_x(\ell)H^*_y(\ell)+H_x(\ell+1)H^*_y(\ell+1)\})$$

$$\mathcal{M}'_\Gamma(H_x(\ell+1)H^*_x(\ell)+H^*_x(\ell+1)H_x(\ell)+H_y(\ell+1)H^*_y(\ell)$$

$$H^*_y(\ell+1)H_y(\ell)+H_x(\ell)H^*_y(\ell+1)+H_x(\ell+1)H^*_y(\ell)))$$

Applying the result of our second lemma (Equation (A.15)), we obtain

$$E_b E_\tau\{|I_\mu - I_\lambda|^2\} \tag{A.19}$$

$$=(1/T)\sum_{\ell=0}^{N-1}(\mathcal{M}_\Gamma\, 4N +$$

$$\mathcal{M}'_\Gamma(2\mathrm{Re}\{C_{x,x}(1)\}+2\mathrm{Re}\{C_{y,y}(1)\}+C^*_{x,y}(-1)+C^*_{x,y}(1)))$$

$$=(1/T_c)(\mathcal{M}_\Gamma 4N +$$

$$\mathcal{M}'_\Gamma(2\mathrm{Re}\{C_{x,x}(1)\}+2\mathrm{Re}\{C_{y,y}(1)\}+C^*_{x,y}(-1)+C^*_{x,y}(1)))$$

## A.5 Second Moment of Real Multiple Access Interference

By applying Equation (A.9) to Equation (A.19), we obtain the following result, which we use in Chapter 3

$$E_b E_\tau\{(I_\mu - I_\lambda)^2\} = (1/8T_c)(\mathcal{M}_\Gamma 4N + \tag{A.20}$$

$$\mathcal{M}'_\Gamma(2\mathrm{Re}\{C_{x_\mu,x_\mu}(1)\}+2\mathrm{Re}\{C_{x_\lambda,x_\lambda}(1)\}+C^*_{x_\mu,x_\lambda}(-1)+C^*_{x_\mu,x_\lambda}(1)))$$

# APPENDIX B

## DERIVATION OF CHARACTERISTIC FUNCTION OF MULTIPLE ACCESS INTERFERENCE

The characteristic function of $I_\mu^{(\beta)}-I_\lambda^{(\beta)}$ is sought where $I_\mu^{(\beta)}$ and $I_\lambda^{(\beta)}$ are the interference terms introduced into the outputs of the $\mu^{th}$ and $\lambda^{th}$ correlators by the $\beta^{th}$ interfering user. We denote

$$I_\mu^{(\beta)}-I_\lambda^{(\beta)}=I_\mu-I_\lambda = \tag{B.1}$$

$$\tfrac{1}{2}\hat{R}(\tau)\ \mathrm{Re}\{\exp j\omega_c\tau(C^*_{x_\mu-x_\lambda,u}(\ell)+C_{v,x_\mu-x_\lambda}(N-\ell))\}+$$

$$\tfrac{1}{2}R(\tau)\ \mathrm{Re}\{\exp j\omega_c\tau(C^*_{x_\mu-x_\lambda,u}(\ell+1)+C_{v,x_\mu-x_\lambda}(N-\ell-1))\}$$

and

$$\phi^{(\beta)}(\omega)=E_\tau^{(\beta)}E_b^{(\beta)}E_c^{(\beta)}E_c^{(\alpha)}\{\exp j\omega(I_\mu-I_\lambda)\} \tag{B.2}$$

The above expressions may be combined and manipulated to provide the following key relation

$$\phi^{(\beta)}(\omega)=E_\tau^{(\beta)}E_b^{(\beta)}E_c^{(\beta)}E_c^{(\alpha)}\{ \tag{B.3}$$

$$\prod_{n=\ell}^{N-1}\exp j\omega\ \tfrac{1}{2}\hat{R}(\tau)\ \mathrm{Re}\{|\hat{x}^*_{\mu,n-\ell}-\hat{x}^*_{\lambda,n-\ell}|a_{n-\ell}\}$$

$$\prod_{n=0}^{\ell-1}\exp j\omega\ \tfrac{1}{2}\hat{R}(\tau)\ \mathrm{Re}\{|\hat{x}^*_{\mu,n+N-\ell}-\hat{x}^*_{\lambda,n+N-\ell}|a_{n+N-\ell}\}$$

$$\prod_{n=\ell+1}^{N-1}\exp j\omega\ \tfrac{1}{2}R(\tau)\ \mathrm{Re}\{|\hat{x}^*_{\mu,n-\ell-1}-\hat{x}^*_{\lambda,n-\ell-1}|a_{n-\ell-1}\}$$

$$\prod_{n=0}^{\ell}\exp j\omega\ \tfrac{1}{2}R(\tau)\ \mathrm{Re}\{|x^*_{\mu,n+N-\ell-1}-x^*_{\lambda,n+N-\ell-1}|a_{n+N-\ell-1}\}\}$$

where the $a_n$ are products as follows

$$a_{n-\ell}=\exp j\omega_c\tau \ \exp j\hat{\theta}_{n-\ell} \ c^{*(\alpha)}_{n-\ell} \ c^{(\beta)}_n \ \hat{u}_n \tag{B.4}$$

$$a_{n+N-\ell}=\exp j\omega_c\tau \ \exp j\hat{\theta}_{n+N-\ell} \ c^{*(\alpha)}_{n+N-\ell} \ c^{(\beta)}_n \ \hat{u}_n \tag{B.5}$$

$$a_{n-\ell-1}=\exp j\omega_c\tau \ \exp j\hat{\theta}_{n-\ell-1} \ c^{*(\alpha)}_{n-\ell-1} \ c^{(\beta)}_n \ \hat{u}_n \tag{B.6}$$

$$a_{n+N-\ell-1}=\exp j\omega_c\tau \ \exp j\hat{\theta}_{n+N-\ell-1} \ c^{*}_{n+N-\ell-1} \ c^{(\beta)}_n \ \hat{v}_n \tag{B.7}$$

The $c_n^{(\alpha)}$ and $c_n^{(\beta)}$ are the random elements of the coset leaders for the $\alpha^{th}$ and $\beta^{th}$ users respectively, hence

$$x_{\mu,n}=\hat{x}_{\mu,n}c_n^{(\alpha)} \tag{B.8}$$

$$b^{(\beta)}=(\hat{u}_0^{(\beta)}c_0^{(\beta)} \ \ldots \ \hat{u}_{N-1}^{(\beta)}c_{N-1}^{(\beta)} \ \ \hat{v}_0^{(\beta)}c_0^{(\beta)} \ \ldots \ \hat{v}_{N-1}^{(\beta)}c_{N-1}^{(\beta)}) \tag{B.9}$$

The $\hat{\theta}_n$ are the phase angles of the differences $\hat{x}_{\mu,n}-\hat{x}_{\lambda,n}$. The $a_n$ are complex random variables with unity magnitude and phase which is uniformly distributed on $[0,2\pi)$. Most importantly, they are independent for all n, hence we may derive

$$\phi^{(\beta)}(\omega)=E_{\tau}\prod_{n=0}^{N-1}(1/2\pi)\int_{0}^{2\pi}\exp j\omega\tfrac{1}{2}\hat{R}(\tau)|\hat{x}_{\mu,n}-\hat{x}_{\lambda,n}|\cos\theta d\theta \qquad (B.10)$$

$$\cdot\prod_{n=0}^{N-1}(1/2\pi)\int_{0}^{2\pi}\exp j\omega\tfrac{1}{2}R(\tau)|\hat{x}_{\mu,n}-\hat{x}_{\lambda,n}|\cos\theta d\theta$$

$$=(1/T_c)\int_{0}^{T_c}\prod_{n=0}^{N-1}J_o(\tfrac{1}{2}\omega\hat{R}(\tau)|\hat{x}_{\mu,n}-\hat{x}_{\lambda,n}|)$$

$$\circ\prod_{n=0}^{N-1}J_o(\tfrac{1}{2}\omega R(\tau)|\hat{x}_{\mu,n}-\hat{x}_{\lambda,n}|)d\tau$$

This expression may be further developed by considering

$$|\hat{x}_{\mu,n}-\hat{x}_{\lambda,n}|=\sqrt{2}\sqrt{1-\cos(\theta_{\mu,n}-\theta_{\lambda,n})} \qquad (B.11)$$

For r-phase orthogonal codes, codewords are orthogonal, because the set of phase angle differences $(\theta_{\mu,n}-\theta_{\lambda,n})$ are evenly spaced over the interval $[0,2\pi)$. Consequently, we may write

$$\phi^{(\beta)}(\omega)=(1/T_c)\int_{0}^{T_c}\prod_{n=0}^{N-1}J_o(\tfrac{1}{2}\omega\hat{R}(\tau)\sqrt{2}\sqrt{1-\cos(2\pi n/N)}) \qquad (B.12)$$

$$\circ\prod_{n=0}^{N-1}J_o(\tfrac{1}{2}\omega R(\tau)\sqrt{2}\sqrt{1-\cos(2\pi n/N)})d\tau$$

This is the expression we use in Section 3.4.3.

# APPENDIX C

## CHARACTERISTIC FUNCTION APPROACH FOR DETERMINISTIC CODES

In the subsections 3.4.3 and 3.4.5, we have used characteristic functions to evaluate the probability of error in systems using some variety of random coding. This technique has resulted in probability expressions that are very easy to evaluate numerically. However, Geraniotis and Pursley (1982) have also used the characteristic function approach with deterministic sequences in their antipodal signalling model. In fact, they have produced algorithms which calculate the probability of error, and the computational requirements of these algorithms only grow <u>linearly</u> with K or N. Unfortunately, similar algorithms for our model are <u>much</u> less efficient. To see why, consider the multiple access characteristic function

$$\phi^{(\beta)}(\omega)=E_{\tau}^{(\beta)}E_{b}^{(\beta)}\{\exp(-j\omega(2/N\varepsilon_{\Gamma})(I_{\mu}^{(\beta)}-I_{\lambda}^{(\beta)}))\} \qquad (C.1)$$

As shown, this function depends on $\mu$ and $\lambda$ and must be averaged over the $M^2$ possible sequence overlaps $b^{(\beta)}$. Consequently, our algorithm has a computational requirement proportional to $KN^5$. Even though this may be reduced to $KN^4$ by judicious program design, the run time of this algorithm is prohibitive for all but the smallest sequence lengths.

We did not use our algorithm to investigate average system performance, but we did use it to provide some results on the worst case probability of error as a function of user delays.

This was done for the following three reasons, which we will expand on below.

- Worst case delay performance is of particular interest for the LF channel.
- The random coding results given earlier will provide good estimates of the average performance of deterministic sets. However, they are not useful for estimating the worst case delay performance of deterministic sets.
- For worst case performance, our algorithm can be made computationally more efficient after making some reasonable assumptions.

Worst case delay performance is of interest, because unfavorable relative signal delays may exist for some LF band users indefinitely. The relative signal delay between two signals will change if the delay due to signal propagation or transmitter timing is a function of time. However, LF communications usually employ a groundwave propagation mode which introduces negligible delay change as a function of time. Additionally, the transmitter timing must be rather well controlled to allow receiver synchronization, and this control prevents rapid relative signal delay change. This latter fact is especially true if the LF signals are also serving a navigation role, which implies extremely stable control of transmitter timing.

The random coding approach employed at the end of Chapter two is not a good one for investigating the worst case probabilities as a function of user delays. For the random coding approach, the maximum probability of error occurs any time the user delays are equal to an integer times the chip width. For actual codes, the probability of error for any set of user delays is a function of the various partial cross correlation functions. Consequently, if the user delays were constrained to integer multiples of the chip width, the resulting probability of error would still vary considerably. However, the random coding approach does provide good estimates when the probability of error is averaged with respect to the user delays.

For worst case performance, our algorithm can be made somewhat more efficient, provided we are willing to make some reasonable assumptions:

- The greatest probability of error occurs when all user delays are integer multiples of a chip width.
- The worst case $\tau^{(\beta)}$ is independent of $\tau^{(\gamma)}$ for all $\beta$ not equal to $\gamma$.
- The worst case delays are independent of $\varepsilon_b/N_o$.

An algorithm employing these assumptions was designed and used to generate the upper curves in Figures C.1 and C.2. These curves are the worst case probability of error as a function of user delays ($\max_{\tau} \Pr(\varepsilon,\tau)$) for the length 17 additive character codes. The lower curves are the average probability of errors given by Equations (3.58) and (3.59). As shown in the figures, the worst case probability of error can be orders of magnitude greater than the average probability of error. However, $\Pr(\varepsilon,\tau)$

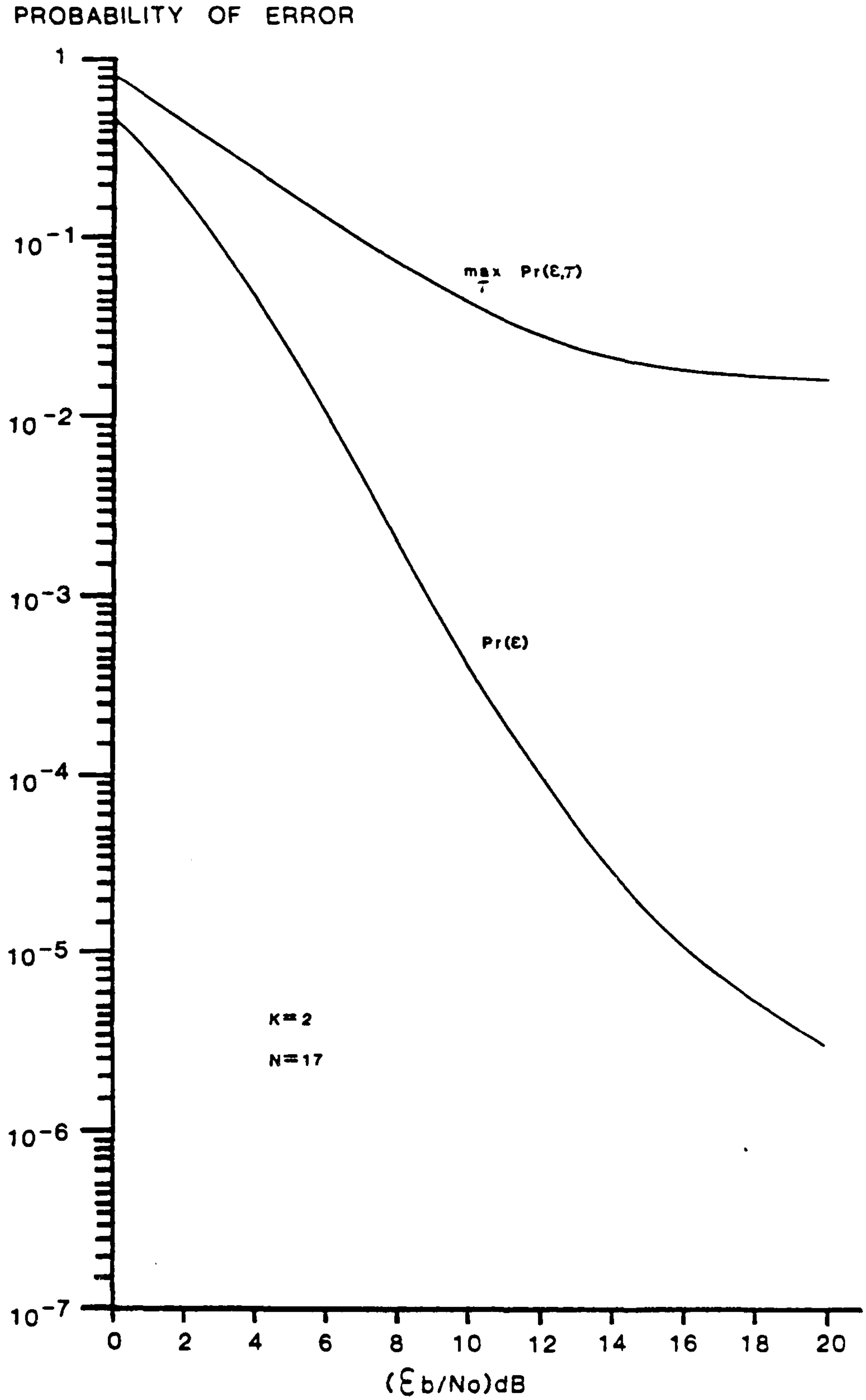


Figure C.1 Worst Case and Average Multiuser Error Probabilities for N=17 Additive Character Sequence Sets

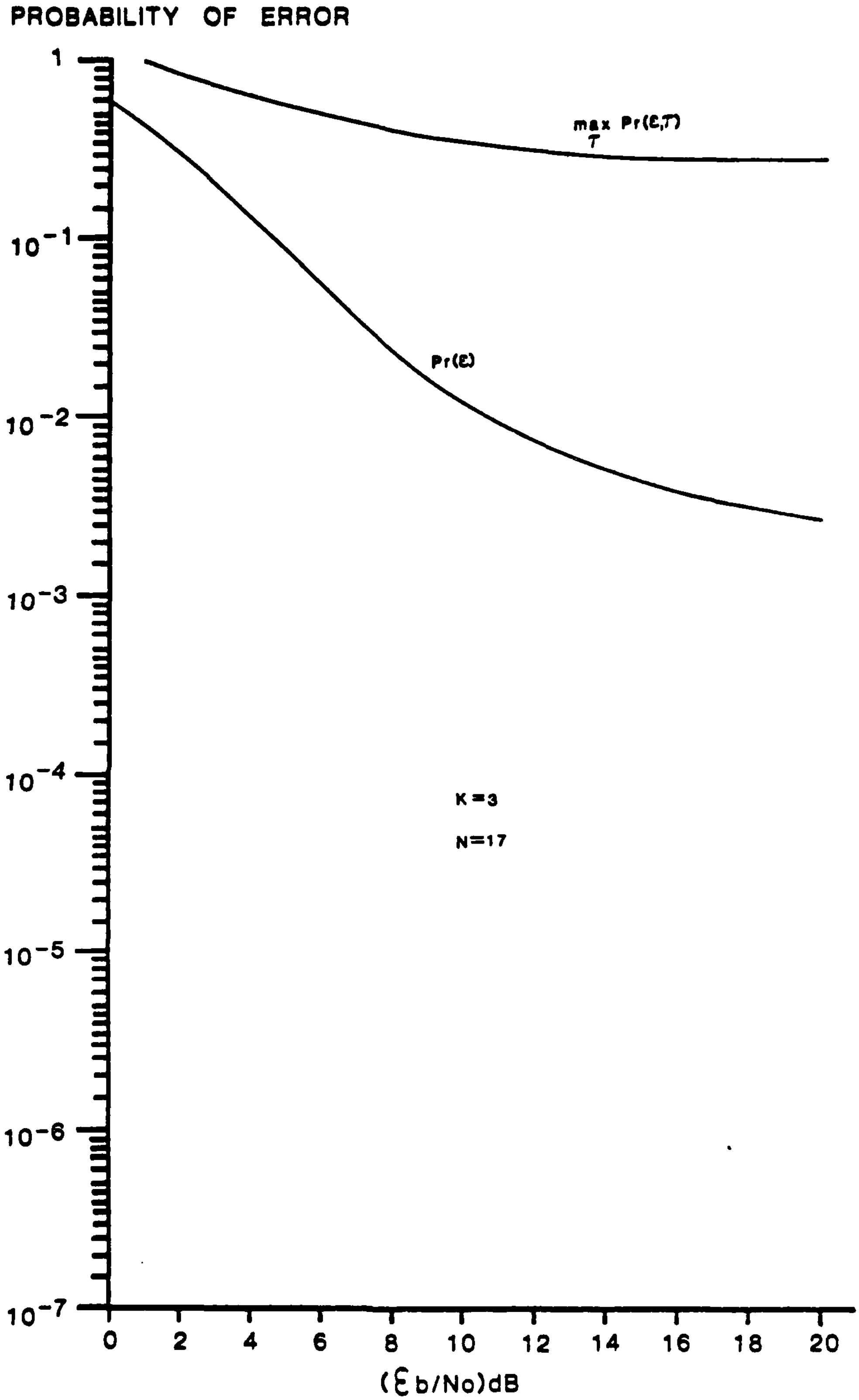


Figure C.2 Worst Case and Average Multiuser Error Probabilities for N=17 Additive Character Sequence Sets is near the worst case curve only for a very small range of user delays. Additionally, a small set of results indicates that the average performance of the additive character sets is very close to the lower curves. Consequently, the average performance curves from Chapter 3 are good design tools whenever the user delays vary even slowly.

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# VITA

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"Direct-Sequence Spread-Spectrum Communication Over the Low Frequency Channel," to appear in the Proceedings of the 1983 Conference on Information Sciences and Systems, Johns Hopkins University, Baltimore, MD. (with Dilip Sarwate)

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